

Homework Assignment 1: Due 3:15pm Tuesday, Oct 6, 2009

There is a homework collecting box outside of professor Ye's office (Terman 316) for you to submit your homework. No late homework accepted!

Problem 1

LLYQ (Leila, Lawrence, Yihan, and Qi) ice cream Co. is producing three flavors of ice cream: vanilla, mint chip, and chocolate. These products are manufactured using common ingredients: milk, eggs, and pre-made powder with unit prices \$5, \$3, and \$10 respectively. The three products also rely on common manufacturing processes: mixing and freezing (The mixing of a batch doesn't have to precede its freezing process, i.e., there isn't a sequential issue for mixing and freezing). The cost is \$1.5 per hour per gallon for mixing, and \$1 per hour per gallon for freezing.

Production of a gallon of vanilla ice cream requires 3 units of milk, 2 units of eggs, 1 unit of pre-made powder, 2 hours of mixing, and 5 hours of freezing. Production of a gallon of mint chip ice cream requires 4 units of milk, 1.5 units of eggs, 0.5 units of pre-made powder, 2 hours of mixing, and 4 hours of freezing. Production of a gallon of chocolate ice cream requires 3.5 units of milk, 1 unit of eggs, 1.5 units of pre-made powder, 1 hour of mixing, and 3 hours of freezing. Each of the three flavors sells for \$80 per gallon.

A total of 1000 units of milk, 500 units of eggs, and 250 units of pre-made powder are delivered to the warehouse each day. The factory is equipped to mix up to 20 gallons per hour and freeze up to 40 gallons per hour, for up to 24 hours per day. A contractual arrangement with a restaurant chain requires that the company produces at least 50 gallons of vanilla ice cream per day. The company's objective is to maximize profit.

- a) Formulate a linear program that can be used to determine how much vanilla, mint chip, and chocolate ice cream the company should produce everyday.
- b) Solve the linear program by Excel Solver (or any Software) and provide the numerical quantities (Just submit your final result).

Problem 2

Consider a school district with I neighborhoods, J schools, and G grades at each school. Each school j has a capacity of C_{jg} for grade g . In each neighborhood i , the student population of grade g is S_{ig} . Finally, the distance of school j from neighborhood i is d_{ij} . Formulate a linear programming problem whose objective is to assign all students to schools, while minimizing the total distance traveled by all students. You may ignore the fact that number of students must be integer.

Problem 3

A function f is called concave if, for each pair of values x_1 and x_2 in its domain, we have $f(\alpha x_1 + (1 - \alpha)x_2) \geq \alpha f(x_1) + (1 - \alpha)f(x_2)$ for all $\alpha \in [0, 1]$.

Let $f(x) = 5 + 2x - x^2$ and $g(x) = 5 - 2x + x^2$. Let $A = \{(x, y) : 1 \leq x \leq 3, 0 \leq y \leq f(x)\}$, $B = \{(x, y) : 1 \leq x \leq 3, 0 \leq y \leq g(x)\}$.

- Is A a convex set? Is B a convex set? Why or why not? (The proof needs a little algebra and patience.)
- Is $f(x)$ a concave function? Is $g(x)$ a concave function? Why or why not?

Problem 4

Samsung has two domestic manufacturing plants for high definition TVs (HDTVs) in Seoul and Busan, two sources of components in Ansan and Gumi, and three main market centers in the world: New York, Tokyo, London. A complete set of components can be used to produce exactly one HDTV. And the transportation cost rates for a complete set of components between the sources and the plants and the transportation cost rates for one HDTV between the plants and the markets are as follows:

		Plant		Market				
		Seoul	Busan	New York	Tokyo	London		
Source	Ansan	\$10	\$15	Plant	Seoul	\$40	\$10	\$20
	Gumi	\$20	\$15		Busan	\$30	\$20	\$30

10,000 complete sets of components for HDTVs are available from Ansan and 15,000 are available from Gumi. We assume the plants have unlimited processing capacity.

- The centers in New York, Tokyo and London have demands 8,000, 14,000 and 3,000 respectively. Formulate the problem of finding the shipping patterns from sources to plants to markets that minimizes the total transportation cost. (You don't have to solve it.)

- b) Now assume each market center requires at least 2,000 Samsung HDTVs. And the profits of selling one HDTV in New York, Tokyo, London are \$120, \$90, \$110 respectively. Formulate the problem to maximize the total net profit (the total market profit minus the transportation cost). (You don't have to solve it.)

Problem 5

Consider the following problem

$$\begin{aligned} \text{Minimize} \quad & 2x_2 + |x_1 - x_3| \\ \text{subject to} \quad & |x_1 + 2| + |x_2| \leq 5 \\ & x_3^2 \leq 1. \end{aligned}$$

Reformulate it as a linear programming problem.

Problem 6

Consider the two-variable linear program with 6 inequality constraints:

$$\begin{aligned} \text{Maximize} \quad & 3x_1 + 5x_2 \\ \text{subject to} \quad & x_1 \geq 0 \\ & x_2 \geq 0 \\ & -x_1 + x_2 \leq 2.5 \\ & x_1 + 2x_2 \leq 9 \\ & x_1 \leq 4 \\ & x_2 \leq 3 \end{aligned}$$

- Plot the constraints in a two-dimensional graph.
- Identify the extreme points of the feasible region.
- Identify the optimal solution point of the problem.

Problem 7

A class of piecewise linear functions can be represented as $f(x) = \text{Maximum}(c_1^T x + d_1, c_2^T x + d_2, \dots, c_p^T x + d_p)$. For such a function f , consider the problem:

$$\begin{aligned} \text{Minimize} \quad & f(x) \\ \text{subject to} \quad & Ax = b \\ & x \geq 0 \end{aligned}$$

Show how to convert this problem to a linear programming problem.