

1. Generate 100 independent copies of $N(0, 1)$ rv's and calculate the average of them.

Solution Matlab commands :

```
>> sum(normrnd(0,1,[1,100]))/100
```

```
ans =
```

```
0.0479
```

2. Let

$$P = \begin{bmatrix} 0 & 0.5 & 0 & 0 & 0.5 \\ 0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0 & 0.5 & 0 \end{bmatrix}.$$

Calculate P^{20} and calculate a row vector π satisfying

$$\pi P = \pi \tag{1}$$

$$\sum_1^5 \pi_i = 1. \tag{2}$$

Solution (1) and (2) can be rewritten as

$$\pi(P - I_5) = \mathbf{0}, \quad \pi \mathbf{e} = 1 \tag{3}$$

where I_5 is a 5 by 5 identity matrix, $\mathbf{0}$ is a 1 by 5 matrix with all zeros, and \mathbf{e} is a 5 by 1 matrix with all ones. Hence, π satisfies

$$\pi \begin{bmatrix} P - I & \mathbf{e} \end{bmatrix} = [\mathbf{0} \ 1].$$

So, we solve the above system of linear equations using Matlab commands:

```
>> P = [0 0.5 0 0 0.5;  
0.5 0 0.5 0 0;  
0 0.5 0 0.5 0;  
0 0 0.5 0 0.5;  
0.5 0 0 0.5 0];
```

```

>> P^20

ans =

    0.2058    0.1953    0.2018    0.2018    0.1953
    0.1953    0.2058    0.1953    0.2018    0.2018
    0.2018    0.1953    0.2058    0.1953    0.2018
    0.2018    0.2018    0.1953    0.2058    0.1953
    0.1953    0.2018    0.2018    0.1953    0.2058
>> A = [P - eye(5), ones(5,1)];
>> b = [zeros(1,5), 1];
>> pi = b/A

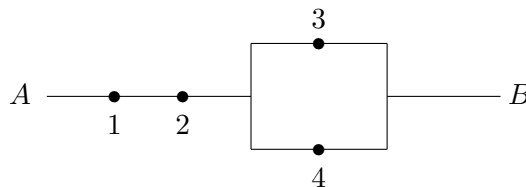
pi =

    0.2000    0.2000    0.2000    0.2000    0.2000

```

3. Suppose that each of the four components in the system below fail independently and have Weibull distributions with shape parameter $1/4$ and scale parameter 1. The system functions so long as A is connected to B .

Use simulation to compute the expected system lifetime.



Solution Let X_i be the lifetime of the i th component for $i = 1, 2, 3,$ and 4 . Then Y , the system lifetime, is $\min\{X_1, X_2, \max\{X_3, X_4\}\}$. Suppose we generate 400 independent copies of Y and calculate the sample average of them to estimate the expected system lifetime. Then, the corresponding Matlab commands are:

```

>> sumOfSamples = 0;
>> for i = 1:400,
    sumOfSamples = sumOfSamples +
        min([weibrnd(1/4,1), weibrnd(1/4,1), max( weibrnd(1/4,1), weibrnd(1/4,1))]);
end
>> sampleMean = sumOfSamples/400

sampleMean =

    1.6014

```