Assignment 8 Solutions

1. Let c and r be the cost for ordering one unit of the book and the price at which the book is sold, respectively. Let y be the inventory level. When the demand exceeds the inventory level, the cost is

$$c(y) = cy + 14 + (c+2)(D-y) - rD$$

and when the inventory level exceeds the demand, the cost is

$$c(y) = cy - (c-1)(y-D) - rD.$$

Hence the cost can be expressed as

$$c(y) = cy + 14I(D>y) + (c+2)(D-y)^+ - (c-1)(y-D)^+ - rD.$$

So, the expected cost is

$$\begin{aligned}
Ec(y) &= cy + 14EI(D>y) + (c+2)E(D-y)^+ - (c-1)E(y-D)^+ - rED \\
&= cy + 14P(D>y) + (c+2)\int_y^{60} (\xi - y)\frac{1}{20}d\xi - (c-1)\int_{40}^{y}(y-\xi)\frac{1}{20}d\xi - 50r \\
&= cy + 14\int_y^{60}\frac{1}{20}d\xi + \frac{(c+2)}{20}[\frac{1}{2}\xi^2 - y\xi]_y^{60} - \frac{(c-1)}{20}[y\xi - \frac{1}{2}\xi^2]_{40}^{y} - 50r \\
&= \frac{3}{40}y^2 - \frac{87}{10}y + 50c - 50r + 262
\end{aligned}$$

and the optimal solution y^* satisfies

$$\frac{dEc(y)}{dy} = \frac{3}{20}y - \frac{87}{10} = 0.$$

Hence $y^* = 58$.

2. (a) The cost function is

$$c(y-x) = c(y-x) + pI(D>y) + hI(y>D).$$

Hence the expected cost is

$$\begin{aligned}
E(c(y-x)) &= c(y-x) + pEI(D>y) \\
&= c(y-x) + pP(D>y) \\
&= c(y-x) + pe^{-y}.
\end{aligned}$$

The optimal inventory level y^* satisfies $\frac{dEc(y-x)}{dy} = c - pe^{-y} = 0$. So $y^* = \ln \frac{p}{c}$.

(b) Let s^* be the point that satisfies $c(s^*) = c(y^*) + K$. Then the optimal policy is to order up to y^* if $x < s^*$ and not to order otherwise.

3. Define $V(i, x)$ by the best expected return on the option given that we haven't exercised the option yet in day i and the stock price is \$$x$. Then

$$\begin{aligned}
V(4,18) &= \max\{3,0\} = \$3 \\
V(4,16) &= \max\{1,0\} = \$1 \\
V(4,14) &= \max\{-1,0\} = \$0 \\
V(4,12) &= \max\{-3,0\} = \$0 \\
V(4,10) &= \max\{-5,0\} = \$0 \\
V(3,17) &= \max\{2, 0.5V(4,18) + 0.5V(4,16)\} = \$2 \\
V(3,15) &= \max\{0, 0.5V(4,16) + 0.5V(4,14)\} = \$0.5 \\
V(3,13) &= \max\{-2, 0.5V(4,14) + 0.5V(4,12)\} = \$0 \\
V(3,11) &= \max\{-4, 0.5V(4,12) + 0.5V(4,10)\} = \$0 \\
V(2,16) &= \max\{1, 0.5V(3,17) + 0.5V(3,15)\} = \$1.25 \\
V(2,14) &= \max\{-1, 0.5V(3,15) + 0.5V(3,13)\} = \$0.25 \\
V(2,12) &= \max\{-3, 0.5V(3,13) + 0.5V(3,11)\} = \$0 \\
V(1,15) &= \max\{0, 0.5V(2,16) + 0.5V(2,14)\} = \$0.75 \\
V(1,13) &= \max\{-2, 0.5V(2,14) + 0.5V(2,12)\} = \$0.125.
\end{aligned}$$

Hence, the current expected value of the option is $0.5V(1,15) + 0.5V(1,13) = 0.4375$.