Assignment 7 - *Due Monday March 9nd*

1. Non-exponential Servers. Suppose that you are the manager of the In-N-Out Burger restaurant and that you are looking to choose between two employees to operate the drive-thru window. The first candidate is Bob for whom the amount of time to complete service with a customer is uniformly distributed between $[0, 3]$ minutes. The second candidate is Charlie who works more slowly, but always completes each service in exact 1.55 minutes. Assume that cars arrive to the drive-thru at an exponential rate of 30 per hour.

   (a) What is the average amount of time a car spends waiting to be served when Bob is working the window? When Charlie is working?

   (b) What is the average number of cars in the drive-thru (in service and waiting) when Bob is working the window? When Charlie is working?

   (c) As the manager, who should you appoint to work the window?

2. Calls are placed to the DMV at a rate of 10 per hour according to a Poisson process. There is only one operator taking calls; arriving callers wait on hold until the operator is available. There is no limit to the number of people that can wait on hold. The length of a call has mean 5 minutes and variance 9 minutes^2.

   (a) What percentage of the time is the operator busy?

   (b) Compute the likelihood that a customer needs to wait for more than 10 minutes.

3. The HP Labs Decision Technology Team consists of 2 members. Each team member works on at most one project at a time, where a project can be either a research project or a consulting project. Each project is staffed by at most one team member. When team members are not working on projects, they find other things to occupy their time until a project is available. If a team member is currently doing a research project, their next project will be a consulting project. Likewise, if a team member is currently doing a consulting project, their next project will be a research project. The duration of a consulting project is exponentially distributed with a mean of 1 month. At most one consulting project can go on in the team at any time, but there is no limit to the number of research projects that can go on at once. Research projects are completed in an exponential length of time with mean 3 months.

   (a) Formulate this stochastic process as a Closed Jackson Network. In particular, identify the customers and the stations. Draw a diagram to illustrate the network, being sure to label the numbers of servers and service rates at each station and the routing probabilities.

   (b) What is the steady state probability that there is one research project and one consulting project going on?

4. In Orlando, Florida, people arrive at Walt Disney's Magic Kingdom according to a Poisson process with rate 1000 people per day. Of the people leaving The Magic Kingdom, 50% will go on to Epcot Center; 10% will go to MGM Studios, and the remaining 40% will go to neither of these places. There are also people who go directly to Epcot Center without

first going to the Magic Kingdom; they arrive according to a Poisson Process with rate 500 per day. People leaving Epcot go to the Magic Kingdom with probability 0.4 and to MGM with probability 0.4. People arrive directly at MGM Studios with exponential interarrival times with rate 200 per day. Those leaving MGM leave the Disney Empire forever (or at least until next spring break.) The times that an individual spends at The Magic Kingdom, Epcot and MGM Studios are exponentially distributed with means 1 day, 1 day and 0.5 days, respectively. There is no limit to the number of people allowed in any of these places.

(a) Formulate this stochastic process as an Open Jackson Network. In particular, identify the customers and the stations. Draw a diagram to illustrate the network, being sure to label the external arrival rates to each station, the number of servers and service rate at each station and the probabilities that dictate customers routes through the network of stations.

(b) Find the total arrival rate (including external and internal arrivals) into each station.

(c) What is the average rate of departure of customers from MGM Studios?

(d) Suppose Disney charges \$40 per day to visit any of their parks. (The charge is prorated according to how long you stay, e.g., the charge for half a day is \$20.) What is Disney's average daily income?

Hint: for this part, you will need the fact that $\sum_{i=0}^{\infty} a^i/i! = e^a$.

(e) Disney Management is considering charging different amounts for the different parks. In particular, the prices that they are debating are \$40 per day for Disney World, \$35 per day for Epcot Center, and \$50 per day for MGM studios. (Again, these charges are prorated.) Assuming that demand is price inelastic (i.e., that these changes will not affect people's patterns of visiting the parks) will this change prove more profitable for them?

5. Consider a single server queue with LIFO (Last-In-First-Out) discipline with nonpreemptive service, that is, a customer who is being served is never interrupted when other customers arrive, and the server always chooses the last person in the queue to be the next to be served. Assume that the queue starts empty at time 0, and suppose that the first eight customers arrive at times $t = 90, 100, 120, 130, 170, 200, 210, 260$ minutes. The service requirements of these first eight customers are: 20 min, 30 min, 30 min, 40 min, 10 min, 50 min, 30 min, 10 min. Compute the number of customers in the queue at time $t = 4$ hours.