Assignment 6 - *Due Monday March 2nd*

1. (H&L 17.5-13) Suppose that a single-server queueing system fits all the assumptions of the birth-and-death process *except* that customers always arrive in *pairs*. The mean arrival rate is 2 pairs per hour (4 customers per hour) and the mean service rate (when the server is busy) is 5 customers per hour.

   (a) Construct the rate diagram for this queueing system.

   (b) Develop the balance equations.

   (c) For comparison purposes, display the rate diagram for the corresponding queueing system that completely fits the birth-and-death process, i.e., where customers arrive *individually* at a mean rate of 4 per hour.

   (d) For the model from part (a), compute the expected number-in-system at time $t = 1$ hr, 10 hr, 100 hr. *Hint:* You will need to use Matlab.

   (e) Compute the expected number-in-system in steady-state for the model of part (a). For this part only, assume that the system has capacity for 10 customers, that is, if the system has 10 customers already in it, then any arriving customers will leave without joining the queue. If the queue has 9 customers and a pair has just arrived, only one of the customers joins the queue.

2. (H&L 17.5-16) Consider a queueing system that has two classes of customers, two clerks providing service, and *no queue*. Potential customers from each class arrive according to a Poisson process, with a mean arrival rate of 10 customers per hour for class 1 and 5 customers per hour for class 2, but these arrivals are lost to the system if they cannot immediately enter service.

   Each customer of class 1 that enters the system will receive service from either one of the clerks that is free, where the service times have an exponential distribution with a mean of 5 minutes.

   Each customer of class 2 that enters the system requires the *simultaneous use of both clerks* (the two clerks work together as a single server), where the service times have an exponential distribution with a mean of 5 minutes. Thus, an arriving customer of this kind would be lost to the system unless both clerks are free to begin service immediately.

   (a) Formulate the queueing model as a continuous time Markov chain by defining the states and construction the rate diagram.

   (b) Now describe how the formulation in part (a) can be fitted into the format of the birth-and-death process.

   (c) Use the results for the birth-and-death process to calculate the steady-state joint distribution of the number of customers of each class in the system.

   (d) For each of the two classes of customers, what is the expected fraction of arrivals who are unable to enter the system?

3. There are two machines, one of which is used as a spare. A working machine will function for an exponential time with rate λ and will then fail. Upon failure, it is immediately replaced by the other machine if that one is in working order, and it goes to the repair facility. The repair facility consists of a single person who takes an exponential time with rate μ to repair a failed machine. At the repair facility, the newly failed machine enters service if the repairperson is free. If the repairperson is busy, it waits until the other machine is fixed; at that time, the newly repaired machine is put in service and repair begins on the other one.

   In the long run, what proportion of time is there a working machine?

4. Customers arrive at a two-server station in accordance with a Poisson process having rate λ. Upon arriving, they join a single queue. Whenever a server completes a service, the person first in line enters service. The service times of server i are exponential with rate μ_i, $i = 1, 2$, where $\mu_1 + \mu_2 > \lambda$. An arrival finding both servers free is equally likely to go to either one. Define an appropriate continuous-time Markov chain for this model and find the limiting probabilities.

   Hint: To compute the steady-state distribution, use the similarity of the model to a standard birth-death process. When 2 or more customers are in the system, the steady state probability for state i can be recursively computed in terms of the steady state probability for state $i-1$.