

Assignment 3 - Due Friday January 30th

1. Suppose that the monthly profit (measured in millions of dollars for a manufacturing facility) is modeled as a first-order autoregressive process ($Y_n : n \geq 0$), where

$$Y_{n+1} = \left(\frac{7}{10}\right)Y_n + Z_{n+1},$$

and ($Z_n : n \geq 1$) is a sequence of independent log-normal rv's with mean 1 and variance 2. The profit last month was \$2 million.

- What do you predict as the total profit over the next 2 months?
 - Compute the mean and variance of the total profit over the next 12 months.
 - What is the likelihood the total profit over the next 12 months will be smaller than \$10 million? (*Hint*: Use the normal approximation)
2. In this problem you will attempt to forecast the price of West Texas Intermediate oil on Wednesday, February 25th, 2009 based on the last Wednesday prices that have been observed since July 11th, 2007. (Why did we pick July 11th? The preceding Wednesday was July 4th, a holiday. To go further back would require that we carefully think through how to deal with the "missing data" from July 4th, 2007. This missing data issue is a common problem in the real-world and comes up in a variety of different settings. For example, what does one do when one is using a company share price as an explanatory variable, and that company merges with another company? Or changes the date on which dividends are paid out?) The Wednesday oil prices since July 11th of 2007 can be found in Table 1 or at

http://www.stanford.edu/class/msande121/handouts/oil_prices_time_series.xls

Use the methods discussed in class to produce a good forecast. (This should include an in-sample, out-of-sample exploration of the data.)

3. Suppose a man possesses r umbrellas that he uses in going from home to his office and back. Each morning he goes to work and returns home in the evening. If it's raining in the morning (resp., in the evening) he takes an umbrella with him to work (resp., home), as long as there is one to take. If it's not raining then he doesn't take an umbrella. Assume that, independent of the past, it rains in the morning (resp., evening) with probability p .
- Define a Markov Chain with $r + 1$ states to help us determine the proportion of time the man gets wet.
 - Show that the steady state probabilities are given by:

$$\pi_i = \begin{cases} \frac{q}{r+q} & \text{if } i = 0 \\ \frac{1}{r+q} & \text{if } i = 1, \dots, r \end{cases}$$

where $q = 1 - p$.

- (c) What fraction of time does the man get wet?
 (d) When $r = 3$, what value of p maximizes the fraction of time he gets wet?

4. Problem 16.6.5, Course Reader

5. Problem 16.7.2, Course Reader

Table 1. *West Texas Intermediate Wednesdays' Oil Prices (7/11/2007 - 1/21/2009)*

Date	Oil Prices	Date	Oil Prices	Date	Oil Prices
7/11/2007	72.58	1/16/2008	90.8	7/23/2008	123.73
7/18/2007	75.03	1/23/2008	87.65	7/30/2008	126.74
7/25/2007	75.74	1/30/2008	92.34	8/6/2008	118.57
8/1/2007	76.49	2/6/2008	87.16	8/13/2008	115.96
8/8/2007	72.23	2/13/2008	93.28	8/20/2008	115.48
8/15/2007	73.36	2/20/2008	100.86	8/27/2008	118.17
8/22/2007	69.3	2/27/2008	99.59	9/3/2008	109.38
8/29/2007	73.52	3/5/2008	104.45	9/10/2008	102.66
9/5/2007	75.74	3/12/2008	109.86	9/17/2008	97.39
9/12/2007	79.85	3/17/2008	103.25	9/25/2008	106.84
9/19/2007	81.99	3/24/2008	105.83	10/1/2008	98.23
9/26/2007	80.31	4/2/2008	104.83	10/8/2008	88.94
10/3/2007	79.97	4/9/2008	110.89	10/15/2008	74.38
10/10/2007	81.3	4/16/2008	114.8	10/22/2008	66.92
10/17/2007	87.19	4/23/2008	119.28	10/29/2008	67.45
10/24/2007	88.3	4/30/2008	113.7	11/5/2008	65.41
10/31/2007	94.16	5/7/2008	123.56	11/12/2008	55.95
11/7/2007	96.46	5/14/2008	124.21	11/19/2008	53.64
11/14/2007	94.02	5/21/2008	132.99	11/26/2008	54.2
11/21/2007	98.57	5/28/2008	131	12/3/2008	46.79
11/28/2007	90.71	6/4/2008	122.3	12/10/2008	43.1
12/5/2007	87.45	6/11/2008	136.43	12/17/2008	40.17
12/12/2007	94.41	6/18/2008	136.54	12/24/2008	32.94
12/19/2007	91.11	6/25/2008	133.92	12/31/2008	44.6
12/26/2007	95.89	7/2/2008	143.74	1/7/2009	42.75
1/2/2008	99.64	7/9/2008	135.88	1/14/2009	37.43
1/9/2008	95.64	7/16/2008	134.63	1/21/2009	42.56