Assignment 2 - *Due Saturday January 24th*

1. Suppose that X represents the proportion of fuel that is left in a car's tank one day after having the tank was last filled. We assume X has a cumulative distribution function (CDF) which is given by x^3 over [0,1]. In this problem, we will see that there are several different ways of generating this random variable.

   (a) Suppose that$M = \max(U_1, U_2, U_3)$, where the U_i's are iid uniform(0,1) rv's. Show that M has the same distribution as X.

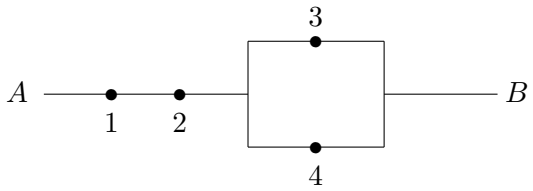
   (b) Discuss how X can be generated by inversion.

   (c) Discuss how X can be generated by acceptance-rejection.

   (d) Suppose that computing the power of a real number takes three times as long as generating a uniform random variable. Which of these methods would be the fastest method for generating X?

2. Suppose that each of the four components in the system below fail independently and have Weibull distributions with shape parameter 1/4 and scale parameter 1. The system functions so long as A is connected to B.

   Use simulation to compute the expected system lifetime. Use a 99% confidence interval to describe the uncertainty in your estimator.



3. Suppose that Ajax Corporation intends to offer a one year warranty on a new product. If the product fails during the warranty period, the customer is given a new item (that carries no warranty). Fifty prototypes of the product have been tested in the company's lab over the last three months. Forty-seven of the prototypes did not fail over the three months. The other three items failed after 0.9 months, 1.2 months and 2.9 months, respectively. Based on previous experience with similar products, the lab believes that the lifetime of the new component can be modeled as an exponential rv.

   (a) If the production cost of each item is expected to be \$20, what is the average cost per item of honoring the warranty?

   (b) What is the likelihood the total warranty cost will be more than \$100000 if the company expects to sell 300000 items?

   (c) Repeat part (a) if a replacement item carries a new one year warranty.

4. Fit a simple linear regression model to China's national income over the period 1952 to 1988 (see Table 1). Predict China's industry national income in 1992.

Table 1. *Real industry national income in China (1952 = 100)*

Year	Industry	Year	Industry
1952	100.0	1971	979.0
1953	133.6	1972	1043.5
1954	159.1	1973	1134.3
1955	169.1	1974	1128.9
1956	219.1	1975	1297.3
1957	244.5	1976	1249.2
1958	383.5	1977	1434.0
1959	501.5	1978	1679.1
1960	541.4	1979	1814.7
1961	315.9	1980	2012.7
1962	267.4	1981	2046.8
1963	300.7	1982	2170.1
1964	374.9	1983	2383.7
1965	477.7	1984	2738.8
1966	598.5	1985	3275.2
1967	504.3	1986	3590.6
1968	458.6	1987	4058.8
1969	622.3	1988	4765.0
1970	863.0		