A (Very) Little Bit About Noise

Rob Candler
April 2007

NOTE: This is a work-in-progress, intended to give some very basic intuition about how to think about noise and bandwidth. Corrections and recommendations are welcome!

Main goal: Get comfortable with the ideas that noise is often given in $\frac{V}{\sqrt{Hz}}$ and you have to multiply by the square root of the bandwidth to get your noise V_{rms}.

After Senturia, Microsystem Design:

Consider that there is some random voltage signal $v_{n(t)}$ (i.e., not a sine wave, but a random looking signal in time with a distribution in frequency).

This signal has a zero value when averaged for all time (i.e., it is bouncing back and forth around zero, but it is doing so randomly).

$$\overline{v}_n = \lim_{t\to\infty} \frac{1}{t} \int_{-t/2}^{t/2} v_n(t)\,dt = 0$$

However, the mean square of the signal is not zero.

$$\overline{v_n^2} = \lim_{t\to\infty} \frac{1}{t} \int_{-t/2}^{t/2} v_n^2(t)\,dt$$

Note #1: Mean square value is simply the average of the square of our voltage.

Note #2: One meaning of the v^2 not being zero is that the power is non-zero. This makes sense; the mean square value of a sine wave is non-zero. This is important, because we use a 60 Hz sine wave to transmit power!

The root-mean-square (rms) of the noise voltage signal, $v_{n,rms}$ is simply the square root of the mean square value, $\overline{v_n^2}$. It is a commonly used metric for the value of a signal. For example, we talk about 120 volts coming from the power outlets in the U.S. It's actually a sine wave with a max amplitude of 170V, but the rms value is 120.

Noise in the frequency domain

Since the noise is random, we can't use typical Fourier methods to represent the noise. We can, however, think about the signal in the frequency domain. We can represent the amplitude of the mean square value, $\overline{v_n^2}$, as a function of frequency. This is called the spectral density, which gives the mean square value *density* as a function of frequency, $S_n(f)$. Note that the units here are V^2/Hz or I^2/Hz, depending on the type of noise.

We can get the mean square value, $\overline{v_n^2}$, *for a specific frequency range* by integrating $S_n(f)$ over the frequency range of interest.

$$\overline{v_n^2} = \int_{f_1}^{f_2} S_n(f)df$$

We can then take the square root to get the root-mean-square value, $v_{n,rms}$.

White noise: the simple case:

The integration of $S_n(f)$ is simple for the case of white noise, where $S_n(f)$ is just a constant value, independent of frequency. This is the case for Johnson (thermal) noise, up to 100 MHz or so. For Johnson noise, the mean square value density is well described by

$$S_n(f) = 4k_bTR$$

where k_b is the Boltzmann constant, T is temperature, and R is the resistance.

Integrating over some frequency range to get the mean square value is simple for this case.

$$\overline{v_n^2} = \int_{f_1}^{f_2} 4k_bTRdf = 4k_bTR(f_2 - f_1) = 4k_bTR(BW)$$

where BW is bandwidth.

The root-mean-square value is therefore

$$v_{n,rms} = \sqrt{4k_bTR(BW)} = \sqrt{4k_bTR}\sqrt{(BW)}$$

This shows why we multiply by the square root of the bandwidth. The spectrum that we integrate is given in terms of V^2, necessitating the square root. As one other piece of intuition, if you divided $S_n(f)$ for Johnson noise by R, you would have a *power* spectral density (V^2/R on the left side, $4k_bT$ on the right, both with units Watts).