ME220 Problem Set 4 Due 5/14/08

1. Footprint Sensor Problem

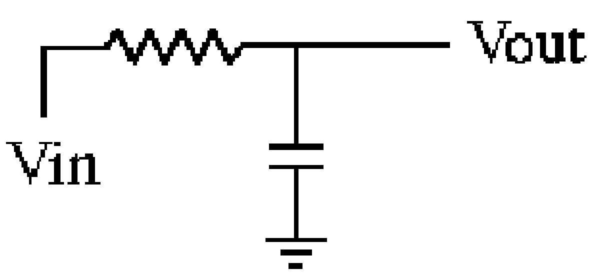
It is Easter weekend and all electronics hardware stores in your town are closed for the holidays. However, a recent report on the increasing crime rate in your area had gotten on your nerves, and you've decided to do something about it right away. Digging out your toolkit from a sensor class that you had taken years ago, you find a lot of capacitive transducers that consist of a pair of electrodes separate by a compressible material layer.

Your idea is to attach a few of these capacitive devices to a long metal strip under the mat right behind your apartment door where every entering person would surely step. You also plan to hook it up with some electronics and to a siren system that hopefully would scare any potential burglars away when anyone steps on it. For now, we're just going to worry about the sensor.

After running some simple experiment in the garage below your apartment with an aged, old computer, (installed with your familiar LabView program), you figure out the following:

a. The capacitive sensor consists of a pair of circular electrodes, each with an area of 100 cm^2. The gap between the electrodes is 1 millimeters. Assume the compressible material layer has a relative permittivity of 20. What is the capacitance?

Your circuit looks something like this:



b. What is the amplitude of Vout if you use a 5V source at 1 kHz as your Vin ? (Assume C is the capacitance from the first question and that R is 100kΩ)

c. Now, you tried to step on the mat with varying weight (using a backpack with varying loads to simulate those 200 lb folks etc), and you determine that the gap, X, has a transfer function with the weight $X(w) = 1-\alpha*w$, where w is the weight in kilograms, X is in kilograms and $\alpha = 0.004$ {mm/kg}. What is the amplitude of Vout if a 70kg guy steps on it and stands there?

You also figure out that a potential burglar would not stand still on your mat. Think about what the typical event would be like, and draw a picture of the signal at Vout as a function of time during a typical event.

2. Smart Piñata (Design Problem)

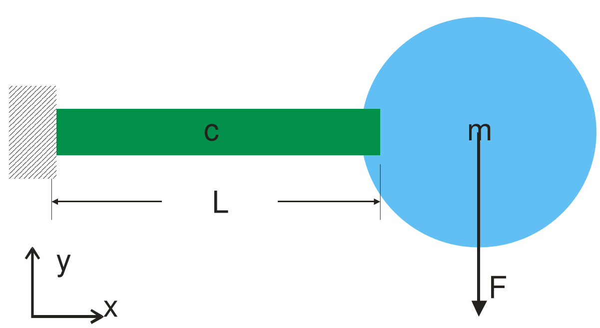
Your daughter's birthday party is just less than 12 hours away & you would like to impress all your guests and their parents with your newly acquired knowledge in sensors. It usually takes you & your spouse 3 hours just to make a decent piñata, but you figure out that every kid would like to swing that bat hitting the piñata and measure the strength of their swing. It would be fun if you could have a re-usable "electronic piñata" with some built in sensors to measure the impact of the bat.

Before picking a sensor from Digikey or Jameco, you need to estimate the signals likely to arise from a bat swinging into contact with the piñata. You need estimates for the mass of the piñata, the mass of the bat, the velocity of the bat before the collision, and other parameters as needed to calculate the maximum acceleration that the piñata is likely to experience, and the range of frequencies likely to be present in the interaction.

Use these calculations to specify the necessary sensor parameters for this "product". Things like sensitivity, range, bandwidth, linearity, accuracy, noise – these are examples of the parameters you need to specify.

In this problem, there are no ANSI standards for the masses, velocities, etc. You will need to make estimates. There are no right answers, but there are physically-reasonable answers. You'll need to explain the reasoning behind your answers.

3. Accelerometer Design: You have been asked to help design an accelerometer with a measurement bandwidth of 1000 Hz using a cantilevered design (see figure). The cantilever (c) has a rectangular cross-section and the proof-mass (m) is 1 gram. The cantilever can be made from single-crystal Silicon [E=190 GPa, ρ =2.3 g/cm^3] or Aluminum [E = 75 GPa, ρ =2.7 g/cm^3], your choice.



Assuming this accelerometer design can be properly modeled and analyzed as a second order system (*i.e.,* a lumped mass-spring-damper, ignoring the mass of the cantilever), please answer the following:

a) What is the amount of stiffness in the spring element such that the natural frequency of the device is 2 kHz. Is this a good natural frequency for an accelerometer that may be used for accelerations of frequencies up to 1 kHz? [Hint: recall $\omega=2\pi f$]
b) Please design the dimensions of the accelerometer cantilever given the stiffness calculated in part (a) such that the mass of the beam is less than 2% of the proof-mass. Also comment on which material (Silicon or Aluminum) you selected and explain why you selected this material. [Hints: This is a design problem so there are multiple solutions. If you want a more specific (constrained) case, consider a square cantilever cross-section. Recall that the deflection of a cantilever can be modeled as a linear spring and watch your units].
c) How much deflection will your designed accelerometer cantilever see subjected to a 10g acceleration ($g = 9.8\ m/s^2$) resulting from an external force (F) (assume that the cantilever is massless)?
d) Assume the requirements are that the settling time of accelerometer must be less than 20 milliseconds, what is the amount of viscous damping (or damping ratio) needed? [Hint: Assume the accelerometer is underdamped]

Equations for reference:

$$\omega = \sqrt{\frac{k}{m}}$$

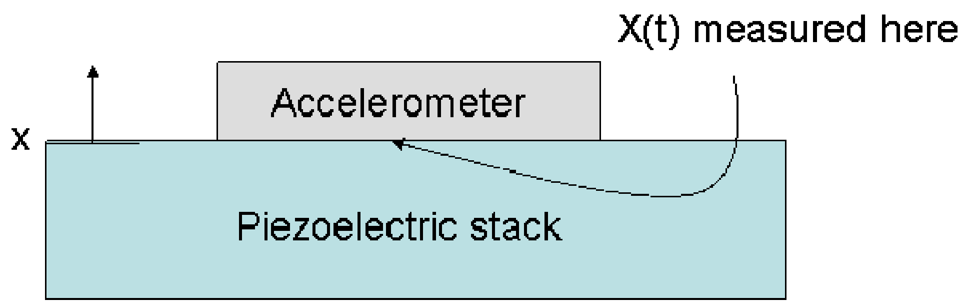
$$\Delta y = \frac{FL^3}{3EI}$$

$$k = \frac{-F}{\Delta y}$$

$$I_{\text{rectangular}} = \frac{bh^3}{12}$$

where: F=Force, E=Young's modulus, L=length, k is the stiffness, and Δy is deflection of the tip of the cantilever

4. Accelerometer Testing Calculations: We have a MMA1201P accelerometer and a BM532 50-layer piezoelectric stack (see class website for datasheet links). We mount the accelerometer on the piezoelectric stack, then drive the stack with a sine wave voltage input of frequency f_0 and measure the response of the accelerometer. Assume that you only look at the frequency component at f_0 of the accelerometer output signal. Also assume that the accelerometer is connected as recommended and powered from a 5V supply.



a) The position of the top of the stack, where the accelerometer is glued, is $x(t) = X_0*\sin(2\pi* f_0*t)$ {m}. Write an expression for the acceleration of that point.
b) The voltage applied to the stack is $V_{stack}(t) = V_{stack0}*\sin(2\pi* f_0*t)$ {V}. Find X_0 in terms of V_{stack0} based on the data plots in the datasheets. Assume the plot is for a stack with 50 layers of piezoelectric material.
c) The expected voltage output of the accelerometer is $V_{acc}(t) = V_{acc0} + \alpha*\sin(2\pi* f_0*t + \phi)$ {V}, where ϕ is a possible phase shift. Find V_{acc0} and α based on the datasheets. Write your answers in terms of V_{stack0}, ϕ, and f_0 . Assume f_0 is $\leq$ 100 Hz. [Hint: for the phase, you want to fit it such that α is positive. Can you incorporate convert a '-' sign into phase, ϕ?]
d) If you drive the stack at 100 Hz, what is the smallest amplitude voltage of V_{stack0} you can use to get a SNR of 5? (Calculate SNR based on the signal amplitude divided by the RMS noise present with the chosen bandwidth). Assume you are sampling at 2kHz without any filtering.
e) You apply an ideal bandpass filter from 50-150Hz to the signal. Does the answer to the last question change? If so, what is the new V_{stack0}?
f) Going back to the set-up of part d, you drive the stack at 50 Hz instead. Does your answer for V_{stack0} change? If so, does it get larger or smaller? By what factor?
g) The piezoelectric stack requires a fairly large current, which means that there will be noise on the ground line at the driving frequency f_0. Since it is at the same frequency as your signal, there is no way to filter it out. This noise is manifest as a variation in the supply voltage V_{DD} seen by the accelerometer. If the ground ripple is 50 mV, how large of an effect will this have on the output of the accelerometer if you are measuring a 2 g acceleration? Give your answer in terms of the measured acceleration (*e.g.,* it looks like the acceleration is 2.1 g instead of 2 g). [Note: ground ripple is a changing voltage on the ground caused by current flow through the non-zero resistance in the ground wire.]
h) Now suppose this same ground noise couples directly onto the output voltage of the accelerometer because you are using the same ground for the measurement as for the piezo drive. If the ground ripple is 50 mV, how large of an effect will this have on the output of the accelerometer if you are measuring a 2 g acceleration? Give your answer in terms of the measured acceleration (i.e. it looks like the acceleration is 2.1 g instead of 2 g).

5. During basketball games, it is occasionally possible for very large men to slam dunk the basketball with great force, leading to vibrations in the position of the goal. Some league officials have become concerned that the increasing size of the players may push the safety limits of the goal, and they want someone to design a system to detect the motion of the goal. You've been hired to solve this problem using one of the three accelerometers (the MXR7202ML Accelerometer operating at 3V, the ADXL311JE Accelerometer operating at 3V, and the Endevco 65-10-100 (data sheets on course website)). The NBA requirement is for detection of vibration amplitudes in the vertical or lateral axis of greater than 1mm. The backboard and support has a resonant frequency of 2 Hz (*i.e.,* after a nasty slam dunk by Shaquille, the basket will "ring down" at 2 Hz).

Notes: Assume the noise density for the Endevco is constant for all frequencies up to 10 kHz. You can choose if you use the Endevco 10 or 100 model.

a) What is the amplitude of the threshold acceleration?

b) What signal would be produced by each of the 3 accelerometers for this acceleration? I'm just interested in the response to stimulus. Don't worry about the offset.

c) Assume that you've built circuits for measuring the signal with bandwidth of 5 Hz. Can each sensor detect this signal? Calculate the signal/noise ratio for each of these 3 sensors.