

Question #1: You have a capacitive sensor that changes capacitance in response to changes in humidity according to the following equation:

$$C = 0.90\mu\text{F} + 0.002\mu\text{F}*(\% \text{ humidity})$$

Thus for 50% humidity, the capacitance would be exactly 1.0 μF . This is the "nominal value" of the capacitance, and the "nominal output" is the output voltage at 50% humidity.

You integrate this capacitor into an LC circuit and measure the resonant frequency to determine the sensor's change in capacitance due to change in humidity. The inductor has an inductance of 10 mH. The resonant frequency, f_{res} , (in Hz) of an LC oscillator is

$$f_{res} = \frac{1}{2\pi\sqrt{LC}}$$

You are a wizard at circuit design and quickly whip up a circuit that measures the resonant frequency and outputs a voltage with the following transfer function:

$$V_{out} = f_{res} \alpha$$

$$\alpha = \frac{1 \text{ mV}}{\text{Hz}}$$

Thus, the physical input signal consists of the relative humidity (0 to 100%) and the system output is a voltage corresponding to the resonant frequency.

The noise in the system comes from the circuit used to measure the resonant frequency, and has the RMS value of 1 mV/sqrt(Hz).

Assume all system elements of the system are ideal and precisely the value specified.

Q: What is the transfer function of the overall sensor system (humidity \rightarrow voltage)? This should be an equation where the output voltage is a function of all the parameters, especially the humidity.

Q: What is the range of possible outputs? What is the offset?

Q: What is the nominal sensitivity of the sensor system using this 10 mH inductor? What is the sensitivity at 75% humidity? At 100% humidity?

Q: Derive the Taylor Series expansion (through the 2nd derivative) of the output signal about the nominal capacitance value ($C_0 = 1 \mu\text{F}$ when RH = 50%). Give your answer both symbolically (in terms of humidity, L, and C_0) and numerically. This is an expansion of the function $V_{out}(\text{humidity})$.

Q: At 75% humidity, what is the ratio of the quadratic term to the linear term? What would we like this ratio to be? Give your answer both symbolically and in terms of the provided component values. Does the ratio get larger or smaller as the humidity gets closer to 50%?

Q: Assume that the output voltage of this system is to be measured with a data acquisition system. If we look at the voltage with an oscilloscope and no additional filtering (assume the oscilloscope has a bandwidth of 100 MHz), how large is the RMS noise?

Q: Now, assume that there is an ideal (perfectly sharp rolloff) low-pass filter at the output with a cutoff frequency of 100 Hz. How big is the RMS noise?

Q: A humidity sensor is usually not sampled at a high rate – even 100 Hz would be a very high sampling rate for a sensor that probably does not respond to humidity changes very quickly. Besides, Humidity is not expected to change quickly. So, think through the requirements for something like a manufacturing process humidity monitor. State requirements for resolution and measurement rate. Determine the maximum allowable value of RMS noise, and determine the cutoff frequency for the lowpass filter.

Question #2: A resistor has a temperature dependence of resistance given by:

$$R(T) = R_{T_0} e^{\frac{\beta(T-T_0)}{T T_0}}$$

In this expression $T_0 = 300$ K, $\beta = 3000$ K, and R_{T_0} is 1000Ω . This is a typical temperature dependence for a Negative Temperature Coefficient (NTC) thermistor, commonly used as a temperature sensor in low-accuracy applications.

We are interested in using this sensor for temperature measurements near 300K (23C). Carry out a Taylor Series Expansion to the second derivative terms, and evaluate the terms of the expansion.

Q: What is the sensitivity of this sensor at 300K?

Q: How large is the second derivative term relative to the other two terms at 350K? at 400K?

Q: If you were the marketing representative for this sensor, you would need to express the response and error in this sensor in the data sheet. For operation over the range from 300K to 400K, define values for the sensitivity, offset, and error in a way that help minimize the error. Hint – you'll probably find that fitting the data at 300K to a straight line is a poor approximation to the data throughout the 300K-400K range, so think about other ways to fit and represent the data.

Bonus question: Name other ways you can think of to measure temperature. Explain any advantages or disadvantages.