ME220 Homework #1 Solutions

Question #1:

Q: What is the transfer function of the overall sensor system (humidity → voltage)? This should be an equation where the output voltage is a function of all the parameters, especially the humidity.

Solution (10 points): This is basically the combination of the functions described above:

Vout = ((1/(2*pi*sqrt(L*[0.9e-6F + 0.002e-6F*(% humidity)]))/ 1000 Hz)

Q: What is the range of possible outputs? What is the offset?

<u>Solution</u> (5 points): Plug in numbers for 0% (I get 1.678 V) and 100% (I get 1.518). This gives a voltage range from 1.678 to 1.518, and the offset would be defined as the output at 0%

Q: What is the nominal sensitivity of the sensor system using this 10 mH inductor? What is the sensitivity at 75% humidity? At 100% humidity?

<u>Solution</u> (10 points): Sensitivity is the derivative of output signal with respect to the input signal. Nominal could be any number of things, including the average slope, which is given by :
(1.518-1.678)/100% = 1.6 mV/%

The more formal definition is to take derivatives of the output function :

$$Sensitivity = \frac{\partial V}{\partial \%} = \frac{\partial}{\partial \%}\left[\frac{1}{2000\pi\sqrt{L((0.9e-6)+(0.002e-6)(\%))}}\right]$$

$$Sensitivity = \frac{1}{2000\pi\sqrt{(L((0.9e-6)+(0.002e-6)(\%)))^3}}\frac{(-0.002e-6)L}{2}$$

Evaluate this expression at 75% and 100%

I get numbers around 1.5e-3 V/%

Q: Derive the Taylor Series expansion (through the 2nd derivative) of the output signal about the nominal capacitance value (Co = 1 µF when RH = 50%). Give your answer both symbolically (in terms of C, L, and Co) and numerically. This is an expansion of the function $V_{out}(C)$.

<u>Solution</u> (15 points): The taylor series expansion around 50% humidity is

$$Voltage(\%) = Voltage(50\%) + (\% - 50)\left.\frac{\partial V}{\partial \%}\right|_{50\%} + \frac{(\% - 50)^2}{2}\left.\frac{\partial^2 V}{\partial \%^2}\right|_{50\%}$$

$$Voltage(\%) = \frac{1}{2000\pi\sqrt{L(1e-6)}} + (\% - 50)\frac{1}{2000\pi\sqrt{(L(e-6))^3}}\frac{(-0.002e-6)L}{2}$$

$$+\frac{(\% - 50)^2}{2}\frac{1}{2000\pi\sqrt{(L(e-6))^5}}\frac{3(0.002e-6L)^2}{4}$$

$$Voltage(\%) = 1.592 - (1.59e-3)(\% - 50) + (4.78e-6)(\% - 50)^2$$

Then, evaluate this expression by inserting numbers for L, etc at 50% (as above), and 75% Humidity

Q: At 75% humidity, what is the ratio of the quadratic term (quadratic in C - Co) to the linear term? What would we like this ratio to be? Give your answer both symbolically and in terms of the provided component values. Does the ratio get larger or smaller as the humidity gets closer to 50%?

<u>Solution (10 points)</u>:

$$Ratio = \frac{\frac{1}{2000\pi\sqrt{(L(1e-6))^5}}\frac{3(0.002e-6L)^2}{4}\frac{(\% - 50)^2}{2}}{\frac{1}{2000\pi\sqrt{(L(1e-6))^3}}\frac{(-0.002e-6)L}{2}(\% - 50)} = \frac{6(\% - 50)}{4000}$$

We'd like this ratio to be as small as possible. It gets bigger as the % humidity changes from 50%.

Q: Assume that the output voltage of this system is to be measured with a data acquisition system. If we look at the voltage with an oscilloscope and no additional filtering (assume the oscilloscope has a bandwidth of 100 MHz), how large is the RMS noise?

<u>Solution</u> (5 points) :

$$Noise_{RMS} = (1mV/\sqrt{Hz})*(\sqrt{Bandwidth})$$

$$Noise_{RMS} = (1mV/\sqrt{Hz})*(\sqrt{100MHz})$$

$$Noise_{RMS} = 10V$$

Q: Now, assume that there is a low-pass filter at the output with a cutoff frequency of 100 Hz. How big is the RMS noise?

<u>Solution (5 points)</u> :

$$Noise_{RMS} = (1mV / \sqrt{Hz}) * (\sqrt{100 Hz})$$
$$Noise_{RMS} = 10mV$$

Q: A humidity sensor is usually not sampled at a high rate – even 100 Hz would be a very high sampling rate for a sensor that probably does not respond to humidity changes very quickly. Besides, Humidity is not expected to change quickly. So, think through the requirements for something like a manufacturing process humidity monitor. State requirements for resolution and measurement rate. Determine the maximum allowable value of RMS noise, and determine the cutoff frequency for the lowpass filter.

<u>Solution</u> (15 points) : Let's think of a commercial manufacturing process that needs 1% control over the humidity, and that it is to be measured once each minute. We need the amplitude of the RMS noise to be smaller than our signal (around 1.6 mV), and we need the bandwidth to be at least 1/60 Hz. If we build a low pass filter with a cutoff at 1/10 Hz, the RMS noise is 0.3 mV, and the signal for a 1% change in humidity is 1.6 mV, so the signal/noise ratio is about 6, and this is fine.

Also acceptable is to assume a resolution and sensitivity, then find the necessary measurement bandwidth.

Question #2: A resistor has a temperature dependence of resistance given by:

$$R(T) = R_{T0} e^{\frac{\beta(T-T_0)}{TT_0}}$$

In this expression T_0 = 300 K, β = 3000 K, and R_{T0} is 1000 Ω. This is a typical temperature dependence for a Negative Temperature Coefficient (NTC) thermistor, commonly used as a temperature sensor in low-accuracy applications.

We are interested in using this sensor for temperature measurements near 300K (23C). Carry out a Taylor Series Expansion to the second derivative terms, and evaluate the terms of the expansion.

<u>Solution (15 points):</u>

$$R(T) = R(T_o) + (T - T_o)\left.\frac{\partial R(T)}{\partial T}\right|_{T_o} + \frac{(T - T_o)^2}{2}\left.\frac{\partial^2 R(T)}{\partial T^2}\right|_{T_o}$$

$$R(T) = R_{To} + (T - T_o)R_{To}Exp\left[\frac{\beta(T - T_0)}{TT_0}\right]\left[\frac{\beta}{T^2}\right]_{To}$$

$$+\frac{(T - T_0)^2}{2}R_{To}Exp\left[\frac{\beta(T - T_0)}{TT_0}\right]\left[\frac{\beta^2}{T^4} - \frac{2\beta}{T^3}\right]_{To}$$

$$R(T) = R_{To} + (T - T_o)R_{To}\frac{\beta}{T_o^{\,2}} + \frac{(T - T_0)^2}{2}R_{To}\left(\frac{\beta^2}{T_o^{\,4}} - \frac{2\beta}{T_o^{\,3}}\right)$$

$$R(T) = 1000\Omega + (T - T_o)33\Omega + \frac{(T - T_0)^2}{2}1\Omega$$

Q: What is the sensitivity of this sensor at 300K?

<u>Solution</u> (10 points):

$$Sensitivity = \frac{\partial R(T)}{\partial T} = R_{To}Exp\left[\frac{\beta(T - To)}{TTo}\right]\left[\frac{\beta}{TTo} - \frac{\beta(T - To)}{T^2To}\right]$$

$$Sensitivity = R(T)\left[\frac{\beta}{T^2}\right] = 33\Omega / C$$

Q: How large is the second derivative term relative to the other two terms at 350K? at 400K?

<u>Solution (10 points):</u>

$$Ratio = \frac{\frac{(T-T_0)^2}{2} R_{To} \left[\frac{\beta^2}{T_o^{\,4}} - \frac{2\beta}{T_o^{\,3}} \right]_{To}}{R_{To} + (T-T_o) R_{To} \left[\frac{\beta}{T_o^{\,2}} \right]_{To}}$$

$$Ratio(350) = \frac{\frac{(50)^2}{2} 0.46\Omega}{1000\Omega + (50)24\Omega} = 0.26$$

$$Ratio(400) = \frac{\frac{(100)^2}{2} 0.25\Omega}{1000\Omega + (T-T_o)19\Omega} = 0.43$$

Q: If you were the marketing representative for this sensor, you would need to express the response and error in this sensor in the data sheet. For operation over the range from 300K to 400K, define values for the sensitivity, offset, and error in a way that help minimize the error. Hint – you'll probably find that fitting the data at 300K to a straight line is a poor approximation to the data throughout the 300K-400K range, so think about other ways to fit and represent the data.

Solution (10 points):

There are many possible answers to this, but the point is to think about how to draw a line through the data that makes the defined error as small as possible. A least-squares fit is a pretty good choice in this case, but it is not always the best choice, since it minimizes the sum of the squares of the errors at all points, rather than minimizing the maximum error. There is a fit called the "Best Fit" which is the line that is drawn as the bisector of the smallest possible rectangle that can circumscribe all the data. For this question, I'm basically happy if the students have thought about the issues and made a decision based on some good reasoning.