

Prelab	Participation	Lab
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Name: \_\_\_\_\_

## 4 Lab: Forcing functions for a 2<sup>nd</sup>-order, linear, ODE

Some *inputs* to dynamic systems result in interesting dynamic *response*. Inputs that excite dynamic systems include periodic inputs (such as a vibrating motor) or one-time disturbances (such as the step displacement or impulse used in previous labs to determine system parameters). This lab investigates *harmonic* (periodic) sinusoidal inputs because

- A rotary motor with an offset mass creates a sinusoidal forcing function
- The 1<sup>st</sup>-term of the Fourier approximation dominates many real periodic forcing functions

The input forcing function is created by a motor with an offset mass. The motor's base is rigidly attached to a cart that moves along a flat horizontal track. The ODE that describes the horizontal displacement  $x$  of the cart is similar to the ODEs associated with the Scotch-yoke and air-conditioner homework problems. The input motor's angular speed  $\Omega$  will be measured by an encoder. The cart's horizontal acceleration  $\ddot{x}$  will be measured using an accelerometer. You will see the affect of  $\Omega$  on  $x$  and  $\ddot{x}$ .



### 4.1 PreLab: Working Model and brainstorming

1. Download the following Working Model simulations from the class website: HarmonicForcingPogoStick.wm2d
2. Run the Working Model simulations.  
Record results on the Working Model PreLab (see back of the book).
3. Complete the in-class Working Model PreLab problem on Harmonic forcing of a mass-spring-damper system
4. Complete homework problem *Harmonic Forcing of a mass-spring-damper system*, parts a, b, c
5. The most difficult parameter to physically measure is  $m/b/k$  (circle one).  
**Explain:** \_\_\_\_\_
6. Loosely speaking, natural frequency  $\omega_n$  is \_\_\_\_\_.
7. The magnitude of the output response is relatively **small/large** when  $\Omega \ll \omega_n$

8. The magnitude of the output response is relatively **small/large** when  $\Omega \approx \omega_n$
9. The magnitude of the output response is relatively **small/large** when  $\Omega \gg \omega_n$

## 4.2 Experimental

The first step in solving an *inhomogeneous* ODE, is to solve the *homogeneous* part of the ODE. Similarly, the first step in determining a system's *forced response* is to determine the system's *unforced* (natural) response. This lab uses hardware similar to Lab 2 (mass/spring cart system) but employs an offset-mass motor that is rigidly attached to the cart.

### 4.2.1 Determine Physical Parameters

Determine the natural frequency  $\omega_n$  and damping ratio  $\zeta$  for horizontal motions  $x$  of the cart *with* the motor attached to it. Hint: You may use the computer A/D system (terminal) to capture data.

$$\omega_n = \text{_____} \frac{\text{rad}}{\text{sec}} \qquad \zeta = \text{_____} \text{ no units}$$

### 4.2.2 Analytical prediction

The steady-state solution (i.e., the long-term behavior) of the *stable*, constant-coefficient, linear, 2<sup>nd</sup>-order, ODE that governs this sinusoidally-forced laboratory system is

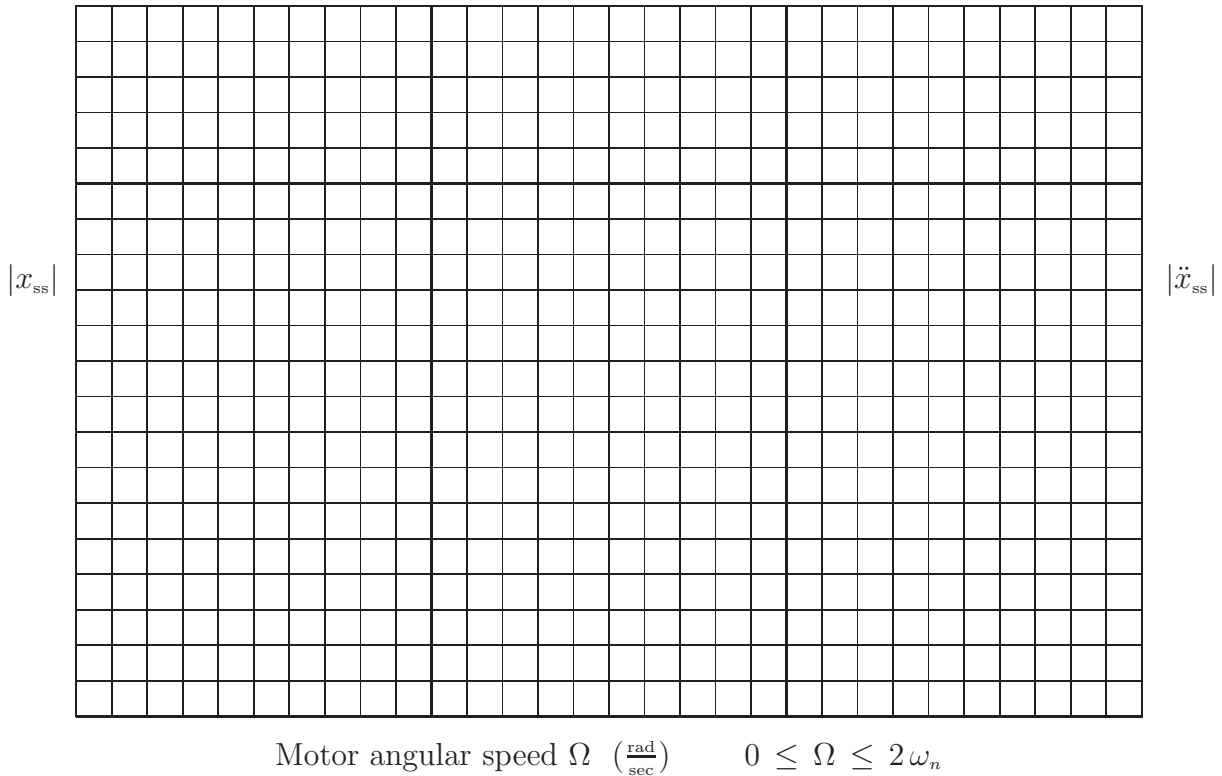
$$x_{ss}(t) = B \sin(\Omega t + \phi) \qquad \text{where} \qquad B = \frac{m_{offset} r / (m_{offset} + m_{cart})}{\sqrt{\left[\left(\frac{\omega_n}{\Omega}\right)^2 - 1\right]^2 + \left[2\zeta\left(\frac{\omega_n}{\Omega}\right)\right]^2}}$$

Determine  $\ddot{x}_{ss}$  and its magnitude  $|\ddot{x}_{ss}|$  in terms of  $B$ ,  $\Omega$ ,  $t$ , and/or  $\phi$ .

$$\ddot{x}_{ss}(t) = \text{_____} \qquad |\ddot{x}_{ss}(t)| = \text{_____}$$

On the following graph,<sup>7</sup> plot  $|x_{ss}(t)|$  for  $0 \leq \Omega \leq 2\omega_n$  (use your experimentally-determined values of  $\omega_n$  and  $\zeta$ ). Similarly, plot  $|\ddot{x}_{ss}(t)|$  versus  $\Omega$ . **Label** the scale on the **left**-axis appropriately for the range of values of  $|x_{ss}(t)|$  and **label** the scale of the **right**-axis for the range of values of  $|\ddot{x}_{ss}(t)|$ . (Note: The left-scale and right-scale will be different). Please show your plots to a TA before moving on.

<sup>7</sup>You may use Excel, Matlab, or MGPlot to plot the graphs. Ensure the scales on **both** the left-axis and right-axis are both labeled.



### 4.2.3 Experimental estimation with an accelerometer

We use several pieces of equipment to measure and record the cart's horizontal acceleration, namely, we use an accelerometer, a microprocessor, a transceiver, and a computer.<sup>8</sup>

- **Accelerometer: ADXL 311 from Analog Devices**

The accelerometer is mounted on a cart and measures acceleration in up to three directions (we use data from only one direction). The accelerometer is relatively small and lightweight as compared to the cart - so its affect on the acceleration of the cart is negligible. The accelerometer's output signal is a linear  $0.3 \frac{\text{volts}}{g}$  signal over a range of  $\pm 2 g$ .<sup>9</sup> The accelerometer is designed to output 2.5 volts when there is **no** acceleration, but there is some variation from one accelerometer to the next. Measure the voltage  $v_{0g}$  output by the accelerometer when the cart is stationary to have all the data needed for the equation relating accelerometer's voltage  $v_{accelerometer}$  to horizontal acceleration  $\ddot{x}$ . Note: To calculate  $v_{0g}$ , use the microprocessor's conversion factor for A/D to accelerometer voltage.

$$v_{0g} = \text{[yellow box]} \text{ volts} \qquad v_{accelerometer} \approx v_{0g} + \left( \frac{0.3 \text{ volts}}{1 g} \right) \ddot{x}$$

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<sup>8</sup>Most accelerometers do not come assembled with a microprocessor, transceiver, and computer.

<sup>9</sup>NIST (National Institute of Standards and Technology) defines  $1 g$  as exactly  $9.80665 \text{ m/sec}^2$ .

- **Breadboard signal processing**

The accelerometer's signal is filter by a low-pass filter to remove high-frequency noise in the signal. Op-amps are used as a buffer to supply additional current and avoid unwanted voltage drops.

- **AVR ATmega168 microprocessor:**

The microprocessor's A/D port receives an analog voltage signal from the breadboard in a specified range (i.e., continuous voltages from 0 volts to 5 volts). The A/D port samples the analog signal at 500 Hz (i.e., at 2 ms intervals). The 10 bit A/D converter on the microprocessor changes the 0 to 5 volt analog signal to bits (ones and zeros) that represent  $2^{10} = 1024$  integer values (e.g., 5 volts converts to 1024 and 2.5 volts converts to 512). Hence,

$$v_{accelerometer} = \frac{5}{1024} * (\text{A/D value})$$

- **DS275 transceiver:**

The transceiver receives bits (ones and zeros) from the microprocessor and translates it to the standard RS-232 serial port communications protocol. This transceiver is somewhat specific to the microprocessor, the serial port, and the communications protocol. For practical purposes, information received by the transceiver is instantaneously translated to the serial port.

- **Computer:**

The computer receives bits from its serial port and uses the software program Teraterm to translate the bits to integer numbers which are then printed to the screen. The numbers displayed on the computer screen are integers from 0 to 1024.

#### 4.2.4 Determining the motor's $\frac{\Omega}{v_{motor}}$ proportionality constant

The maximum (peak) of  $|x_{ss}|$  versus  $\Omega$  should occur at  $\Omega \approx \omega_n$  (you previously determined  $\omega_n$ ). Determine the linear-proportional constant<sup>10</sup> that relates motor angular speed  $\Omega$  to motor input voltage  $v_{motor}$ .

$$\Omega \left( \frac{\text{rad}}{\text{sec}} \right) = \text{[yellow box]} * v_{motor} (\text{rad/s})$$

To complete the previous equation, use an oscilloscope to measure the value of  $v_{motor}$  that corresponds to the **maximum** (peak) value of  $|x_{ss}|$  before breaking out of **frequency locking**.

What is **frequency locking**? (Hint: Ask a TA).

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<sup>10</sup>This **assumes** a directly proportional relationship between  $\Omega$  and  $v_{motor}$ .

### 4.2.5 Data acquisition

1. **Before** data acquisition, complete the  $\Omega$  and  $v_{motor}$  columns of the table below.
2. Login to the computer. Username: me161student. Password: 1euler1
3. Make sure domain says ENGR
4. From the desktop, open TeraTerm Pro (ttermpro)
5. Click the button for serial connection Com1
6. Go to File→Start Log and make a new log file in your group’s folder
7. Turn on the power supply.
8. Have one group member hold the cart
9. Change the voltage to a value specified in the  $v_{motor}$  column of the table below.  
Use the oscilloscope to measure the motor’s input voltage.  
**Do not run the motor at above 4 volts!!**
10. From the terminal screen a menu should appear.  
Choose the “Accelerometer data dump” option. (Press “2”).
11. Release the cart. Note: At low voltages, you may have to nudge the offset mass to overcome static friction and get the motor rotating.
12. Press the space bar to stop recording data
13. Plot the data (e.g., using Excel, Matlab, or **MGPlot**).  
You may have to close the TeraTerm Pro application to close the data file.
14. Find the maximum value of the computer A/D value and convert it to volts.  
Record this value under  $|v_{accelerometer}|$ . Using  $v_{0g}$ , convert  $|v_{accelerometer}|$  to  $|\ddot{x}_{ss}|$ .
15. Repeat steps 8 - 14. to record  $|\ddot{x}_{ss}|$  for  $0 \leq \frac{\Omega}{\omega_n} \leq 2$  in the table below.
16. Ensure the power to the board is off and the setup is neat for the next lab.

Approx. $\frac{\Omega}{\omega_n}$	$\Omega$ (rad/sec)	Motor $v_{motor}$ (volts)	Sensor $ v_{accelerometer} $ (volts)	$ \ddot{x}_{ss} $ ( $m/sec^2$ )
0.50				
0.70				
0.80				
0.90				
1.00				
1.10				
1.20				
1.30				
1.50				
1.70				
1.90				

Plot your experimental data for  $|\ddot{x}_{ss}|$  on your previous graph of analytical  $|\ddot{x}_{ss}|$  versus  $\Omega$ . How well does the analytic model compare to the experimental data? (Circle one)

Great < 0.1%    Very good < 1%    Good < 10%    Fair < 50%    Poor < 100%

Identify potential sources of experimental errors.

- Errors in the physical plant include:
- Errors in the mechatronics include:
- Other errors include: