

Prelab	Participation	Lab
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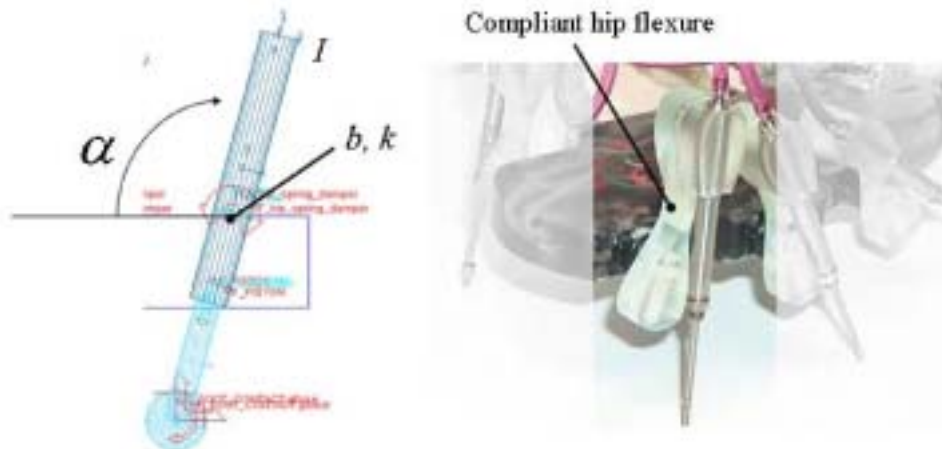
Name: _____

3 Lab 3: Rotational Systems

3.1 Introduction

The Sprawl family of robots has been designed over the past ten years at Stanford University. They are small, hexapedal dynamic running robots capable of speeds of over $5 \frac{\text{body-lengths}}{\text{second}}$ or about $0.8 \frac{\text{m}}{\text{s}}$. They operate without a feedback control system, instead relying on a well-tuned physical system (set of legs) to move quickly and stably, even over obstacles or rough terrain.

During this lab you will be looking at the rotational leg design, and how to tune the leg for the swing phase of locomotion. During normal operation, each full stride of the robot takes 80 ms. The leg is in contact with the ground (stance phase) for about 25 ms. This leaves approximately 55 ms for the leg to retract and swing forward in preparation for the next step. Experience has shown that making the legs too stiff results in slowing the robot down, while making the legs too soft results in crashes. For this lab you will be experimenting with a wooden “leg” that approximates a Sparwilita leg that might be used on a Sprawl robot.



$$I_{yy} \ddot{\theta} + b \dot{\theta} + k \theta - m g L \sin(\theta) = 0$$

3.2 PreLab: Working Model and brainstorming

1. Download the following Working Model simulations from the class website:
MetronomeWnZeta.wm2d
MetronomeIBK.wm2d
MetronomeNonlinearAngle.wm2d
2. Run the Working Model simulations.
Record results on the Working Model PreLab (see back of the book).

3.3 Experimental

The system that you will be evaluating rotates using a compliant joint. The joint was designed and fabricated (from a soft grade of urethane) to be used as a leg flexure for Sprawlita. The leg flexure can be modeled as a pin joint with a built-in rotational spring and damper. It will be your job to experimentally determine the effective stiffness and damping of this joint.

3.3.1 Analytical determination of moment of inertia of a solid block

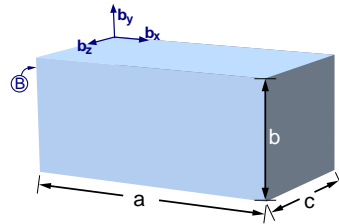
The moment of inertia of a body is defined with a line (axis). The line is usually defined by a specific point on the body and the orientation of the line, Changing the specified point on the body or the line's orientation changes the associated moment of inertia.

The moment of inertia of a uniform rectangular block about the y-axis passing through the block's mass center is:

$$I_{yy} = \frac{1}{12} m (a^2 + c^2)$$

Note: The mass of the object in grams is written on top of the object

Mass of block $m = \underline{\hspace{2cm}}$ kg
Length of block $a = \underline{\hspace{2cm}}$ m
Height of block $b = \underline{\hspace{2cm}}$ m
Width of block $c = \underline{\hspace{2cm}}$ m



Does the height of the block matter in determining I_{yy} ? **Yes/No.**

Explain: Moment of inertia depends on mass-distribution **away from/along** a line.

3.3.2 Methods for determining moments of inertia

Moments of inertia affect the rotational motions of objects and are useful in human-body studies, guidance and control algorithms for aircraft and spacecraft, and the design and control of robotic systems.

Determine a few methods for determining (e.g., calculating, measuring, or approximating) a moment of inertia of a large irregularly-shaped, mostly rigid-body such as an airplane.

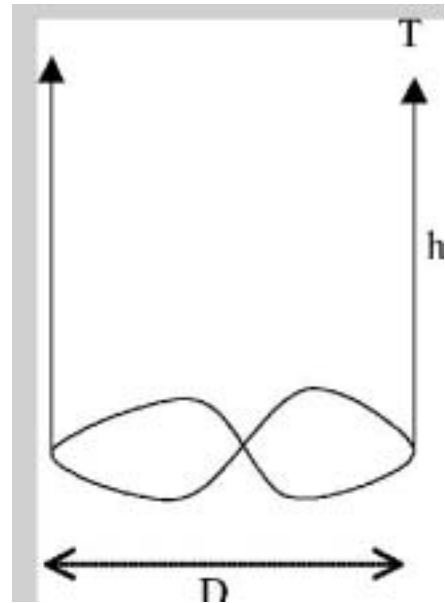
- Compute:
- Experiment:
- Approximate:

3.3.3 Experimental apparatus

- h Length of each suspension wire
- D Distance between (2 parallel) wires
- mg Weight of the object
- I_{yy} Mass moment of inertia about y-axis
- T Tension in string
- θ Angle of rotation

As the object rotates in the horizontal plane, it moves up and down a small but measurable amount. The restoring moments are

$$\Sigma M_y = -2T \sin \theta \frac{D}{2}$$



The relevant equation of motion is shown below. The equation is then simplified using geometry and the small angle approximation $\sin(\theta) \approx \theta$.

$$\Sigma M_y = I_{yy} \ddot{\theta} \quad \Rightarrow \quad -T \sin(\theta) D = I_{yy} \ddot{\theta} \quad \Rightarrow \quad \ddot{\theta} + \frac{mg D^2}{4 I_{yy} h} \theta = 0$$

Using our knowledge of ODEs and rearranging to solve for the moment of inertia gives

$$\omega_n = \sqrt{\frac{mg D^2}{4 I_{yy} h}} \quad \Rightarrow \quad \tau_{period} = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{4 I_{yy} h}{mg D^2}} \quad \Rightarrow \quad I_{yy} = \frac{mg D^2}{16 \pi^2 h} \tau_{period}^2$$

Follow the bifilar pendulum method and record measurements below. Hang the block on two strings (ensure they are **equal length** and **parallel**). Use a stopwatch to determine the system's period of vibrations. Provide an initial **small angle** of $\approx 30^\circ$.

Mass of block	$m =$ _____ kg
Length of the wires	$h =$ _____ m
Distance between wires	$D =$ _____ m
Period of vibrations	$\tau_{period} =$ _____ sec
Moment of inertia	$I_{yy} =$ _____ kg m ²

How well do the predicted and measured value agree? (Circle one)

Note: If you got more than 10% deviation, you probably did something wrong.

- Extremely well (less than 0.1% deviation)
- Very well (less than 1% deviation)
- Good (less than 10% deviation)
- Sort of (less than 50% deviation)
- Poorly (less than 100% deviation)

Using the parallel axis theorem and the moment of inertia determined via the bifilar pendulum experiment, calculate the wooden leg's (B) moment of inertia about the joint. Hint: Use the parallel axis theorem, B 's moment of inertia about its center of mass B_{cm} , the mass of the block, and d , the distance from the pivot to B_{cm} .

$$I_{yy}^{B/joint} = I_{yy}^{B/B_{cm}} + m d^2 = \underline{\hspace{2cm}} \text{ kg m}^2$$

3.3.4 Characterization of the Compliant Joint

Carefully assemble the leg by sliding the joint into the base. Tape the accelerometer to the top of the leg. When you displace the leg, please do so gently. Keep in mind that we are using a small angle approximation when you are thinking about how much to displace the leg. For this part of the lab we will again be using an ADXL 311 accelerometer from Analog Devices to measure the motion of the system. The data acquisition proceeds in a manner similar to lab 2, sampling again at 500Hz.

1. Login to the computer. Username: me161student. Password: 1euler1
2. Make sure domain says ENGR
3. From the desktop, open TeraTerm Pro (ttermpro)
4. Click the button for serial connection Com1
5. Go to File→Start Log and make a new log file in your group's folder
6. Turn on the power supply.
7. On the terminal screen a menu should appear.
Choose the "Accelerometer data dump" option. (Press "2").
8. Have one group member pull the leg slightly to one side and then release.
9. Press the space bar to stop recording data after the cart has stopped.
10. Plot the data (e.g., using Excel, Matlab, or MGPlot).
11. Email the data files and/or graphs to yourself and your group members.
12. Pass in a printed graph of the cart's acceleration ($\frac{m}{sec^2}$) vs. time (sec) with your lab.
13. Ensure the power to the board is off and the setup is neat for the next lab.

3.3.5 Questions

When the small angle approximation $\sin(\theta) \approx \theta$ is used, the equation of motion for small values of θ is

$$I_{yy} \ddot{\theta} + b \dot{\theta} + (k - m g L) \theta = 0$$

Determine the associated expressions of ω_n and ζ

$$\omega_n = \qquad \qquad \qquad \zeta =$$

Use the graph you generated in Section 3.3.4 to determine values for the period of vibration and the decay ratio. (Show work!)

$$\tau_{period} = \text{_____secs} \quad decayRatio = \text{_____no units}$$

Determine numerical values for the natural frequency and damping ratio of the system.

$$\omega_n = \text{_____} \frac{\text{rad}}{\text{sec}} \quad \zeta = \text{_____no units}$$

Determine the physical parameters (I_{yy} , b , k) for this system?

$$I_{yy} = \text{_____} \text{kg m}^2 \quad b = \text{_____} \text{n m sec} \quad k = \text{_____} \frac{\text{n m}}{\text{rad}}$$

† Using your value for the wooden leg's mass moment of inertia (I_{yy}) and experimentally determined value of the flexure stiffness k determine the damping constant b , needed to ensure a settling time of less than 55 ms.

$$k = \text{_____} \frac{\text{n}}{\text{m}}$$