

Prelab √+ √ √- 0	Participation √+ √ √- 0	Lab √+ √ √- 0
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Name: _____

2 Lab: 2nd Order Systems

2.1 Introduction

The first lab investigated motors and 1st-order ODEs. This lab looks at modeling vibrating systems and 2nd-order ODEs of the form

$$m \frac{d^2y}{dt^2} + b \frac{dy}{dt} + k y = 0$$

where the non-zero constants m , b , and k are called the system's *effective mass*, *effective viscous damping*, and *effective spring constant*. Mathematically, the system behavior is more easily described with the following ODE which has only two constants, namely, ζ and ω_n .

$$\frac{d^2y}{dt^2} + 2\zeta\omega_n \frac{dy}{dt} + \omega_n^2 y = 0$$

2.2 PreLab: Motor spin-down (analytical and Working Model)

1. Do the following two homework problems (from the book).
 Problem 2.8: System response and damping
 Problem 2.9: System response for various of ζ
 Problem 2.13: Calculating ζ and ω_n from empirical data for an underdamped 2nd-order ODE
2. Download the following Working Model simulations from the class website:
 CarSuspensionWithWnAndZeta.wm2d
 CarSuspensionWithSpringDamper.wm2d
3. Run the Working Model simulations.
 Record results on the Working Model2 PreLab (see back of the book).

2.3 Experimental

The purpose of this section is to give you a feel for how well real systems can be modeled with 2nd-order ODEs. First you will look at a simple system (a slinky) similar to that discussed in Chapter 7 of the textbook. Then you will characterize the physical parameters of the horizontal cart system that will be used in subsequent labs.

2.3.1 Slinky Revisited

Instead of a slinky, we will use a spring and relatively large mass. This will, hopefully, increase the accuracy of our assumption that the spring is massless. Before you begin the experiment, answer the following questions:

Given the (lumped) mass of the system is 150g, what is the spring constant k ? (Show data taken and calculations. Use SI for all answers)

$$k = \underline{\hspace{2cm}} \frac{N}{m}$$

Now calculate the system's (analytical) natural frequency.

$$\omega_n = \underline{\hspace{2cm}} \frac{rad}{sec}$$

Use a stopwatch to determine the damped natural frequency ω_d of the system.

Hint: Taking an average of a few runs will probably improve your accuracy.

$$\omega_d = \underline{\hspace{2cm}} \frac{rad}{sec}$$

How well do the predicted and measured value agree? (Circle one)

Extremely well (less than 0.1% deviation)

Very well (less than 1% deviation)

Good (less than 10% deviation)

Sort of (less than 50% deviation)

Poorly (less than 100% deviation)

Use a stopwatch to estimate the 1% settling time ($t_{settlng}$) for the system.

Hint: Just eyeball it.

$$t_{settlng} = \underline{\hspace{2cm}} \text{ sec}$$

Knowing $t_s = \frac{4.6}{\zeta \omega_n}$, find the value of ζ .²

$$\zeta = \underline{\hspace{1cm}} \text{ no Units}$$

²The value of ζ is calculated with an experimental value of t_s and a calculated value of ω_n .

† What would the mass of the system need to be to get a settling time of less than 10 seconds? Remember that ζ is a function of m , b , and k .

$$m_{new} = \underline{\hspace{2cm}} \text{ kg}$$

2.3.2 Estimation with an accelerometer

We use several pieces of equipment to measure and record the cart's horizontal acceleration, namely, we use an accelerometer, a microprocessor, a transceiver, and a computer.³

- **Accelerometer: ADXL 311 from Analog Devices**

The accelerometer is mounted on a cart and measures acceleration in up to three directions (we use data from only one direction). The accelerometer is relatively small and lightweight as compared to the cart - so its affect on the acceleration of the cart is negligible. The accelerometer's output signal is a linear $0.3 \frac{\text{volts}}{g}$ signal over a range of ± 2 g. The accelerometer is designed to output 2.5 volts when there is **no** acceleration, but there is some variation from one accelerometer to the next.

- **Breadboard signal processing**

The accelerometer's signal is filter by a low-pass filter to remove high-frequency noise in the signal. Op-amps are used as a buffer to supply additional current and avoid unwanted voltage drops.

- **AVR ATmega168 microprocessor:**

The microprocessor's A/D port receives an analog voltage signal from the breadboard in a specified range (i.e., continuous voltages from 0 volts to 5 volts). The A/D port samples the analog signal at 500 Hz (i.e., at 2 ms intervals). The 10 bit A/D converter on the microprocessor changes the 0 to 5 volt analog signal to bits (ones and zeros) that represent $2^{10} = 1024$ integer values (e.g., 5 volts converts to 1024 and 2.5 volts converts to 512).

- **DS275 transceiver:**

The transceiver receives bits (ones and zeros) from the microprocessor and translates it to the standard RS-232 serial port communications protocol. This transceiver is somewhat specific to the microprocessor, the serial port, and the communications protocol. For practical purposes, information received by the transceiver is instantaneously translated to the serial port.

³Most accelerometers do not come assembled with a microprocessor, transceiver, and computer.

- **Computer:**

The computer receives bits from its serial port and uses the software program Teraterm to translate the bits to integer numbers which are then printed to the screen. The numbers displayed on the computer screen are integers from 0 to 1024.

Data acquisition proceeds in a manner similar to Lab 1.⁴

1. Login to the computer. Username: me161student. Password: 1euler1
2. Make sure domain says ENGR
3. From the desktop, open TeraTerm Pro (ttermpro)
4. Click the button for serial connection Com1
5. Go to File→Start Log and make a new log file in your group's folder
6. Turn on the power supply.
7. On the terminal screen a menu should appear.
Choose the "Accelerometer data dump" option. (Press "2").
8. Have one group member pull the cart to one side and then release it.
9. Press the space bar to stop recording data after the cart has stopped.
10. Plot the data (e.g., using Excel, Matlab, or MGPlot).
11. Email the data files and/or graphs to yourself and your group members.
12. Pass in a printed graph of the cart's acceleration ($\frac{m}{sec^2}$) vs. time (sec) with your lab.
13. Ensure the power to the board is off and the setup is neat for the next lab.

2.3.3 Questions

Use your graph to determine numerical values for the period of vibration and decay ratio. (Show your work!)⁵

$$decayRatio = \text{_____ no units} \qquad \tau_{period} = \text{_____ SECS}$$

⁴The output signal can be double integrated to generate position data, but since this is not necessary to capture the information we are interested in, it will be left as an exercise for the interested student.

⁵The textbook chapter on time-specifications for 2^{nd} -order ODEs is helpful for determining period of vibration and decay ratio.

Calculate the values for the natural frequency and damping ratio of the system.

$$\omega_n = \underline{\hspace{2cm}} \frac{rad}{sec} \quad \zeta = \underline{\hspace{2cm}} \text{ no Units}$$

List three things that could improve the accuracy of these values.

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Now add the mass to the platform and repeat the experiment.

1. Pass in a new graph of the cart's acceleration ($\frac{m}{sec}$) vs. time (sec) with your lab.
2. Calculate the new natural frequency and damping ratio

$$\omega_n = \underline{\hspace{2cm}} \frac{rad}{sec} \quad \zeta = \underline{\hspace{2cm}} \text{ no Units}$$

3. Adding mass **increases/decreases** the natural frequency because $\omega_n = \underline{\hspace{2cm}}$.
4. Adding mass **increases/decreases** the damping ratio because $\zeta = \underline{\hspace{2cm}}$.

†What are the physical parameters (m,b,k) for this system without the added mass? Show your work on a separate page. You will use these values again in later labs!

$$m = \underline{\hspace{2cm}} \text{ kg} \quad b = \underline{\hspace{2cm}} \frac{N \text{ sec}}{m} \quad k = \underline{\hspace{2cm}} \frac{N}{m}$$