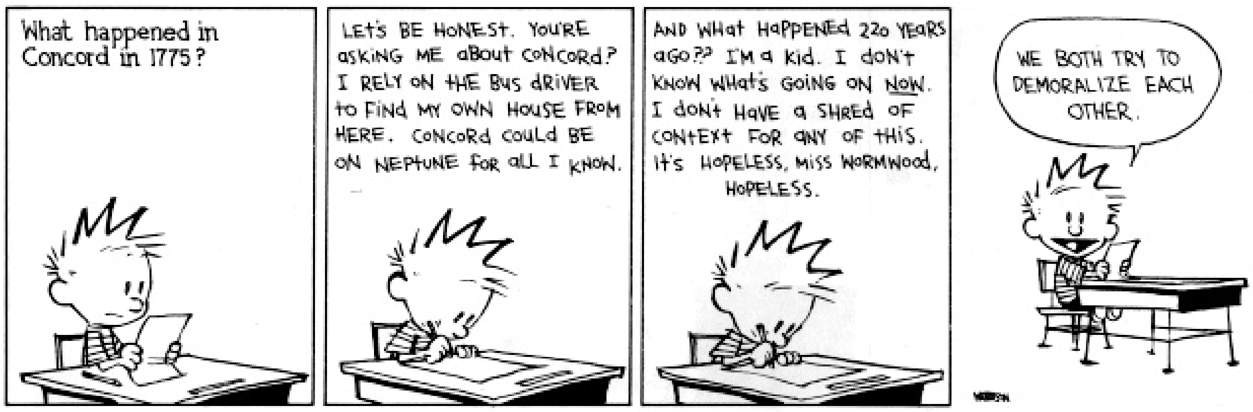
NAME ______________________________

ME161 Midterm. Tuesday October 26, 2004 1:15-2:45 p.m.

I certify that I upheld the Stanford Honor code during this exam ____________________



- Print your name and sign the honor code statement
- You may use your course notes, homework, books, etc.
- Write your answers on this handout
- Where space is provided, show your work to get full credit
- If necessary, attach extra pages for scratch work

Problem	Value	Score
1	26	
2	38	
3	36	
Total	100	

Midterm.1 (26 pts.) True/False, Multiple Choice, Definitions, etc.

(a) **(1 pt.)** Formulas involving *subtraction*, e.g., $\cos(a-b) = \cos(a)*\cos(b) - \sin(a)*\sin(b)$, can always be derived using *negation* and *addition*, e.g., $\cos(a + {}^{-}b) = \cos(a)*\cos({}^{-}b) + \sin(a)*\sin({}^{-}b)$. **True/False**. If your answer is false, provide a counter-example.

(b) **(2 pts.)** For all values of x on your calculator, $\sqrt{x^2} = x$. **True/False**.
If your answer is false, provide a counter-example.

(c) **(3 pts.)** Write the following expression in terms of the sine and cosine function.

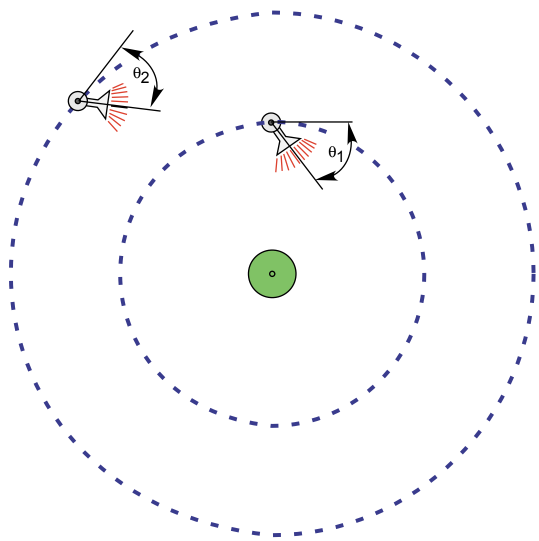
$$e^{({}^{-}3+5\,i)\,*\,t} \;=$$

(d) **(5 pts.)** Put the following function into amplitude/phase form by filling in the blanks.

$$3 * \sin(2t) \;-\; 4 * \cos(2t + \frac{\pi}{3}) \;=\; \qquad\qquad * \cos(2t + \qquad\qquad)$$

(e) **(5 pts.) Minimum fuel-use orbit transfer**

To thrust a satellite from low circular orbit about Earth to a higher circular orbit, an impulse is provided at two instants. The first impulse can be directed radially outward, tangent to the satellite's circular orbit, or directed at some angle θ_1 from the satellite's orbital tangent. The second impulse is applied at apogee (when the satellite is furthest from Earth) and is directed at an angle θ_2 from the orbital tangent.



The first impulse puts the satellite into an elliptical orbit and the second changes the orbit from elliptical to circular.

Using your engineering insights, provide values for θ_1 and θ_2 that minimize the amount of fuel required for this orbital transfer. Explain your reason for choosing these values.

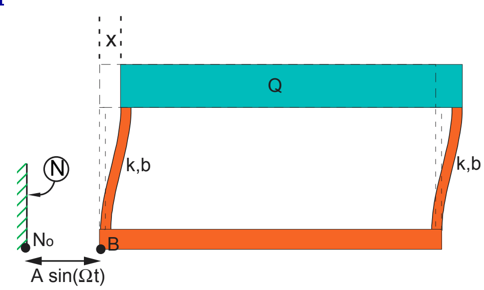
Result:

$\theta_1 = \quad ^\circ \qquad\qquad \theta_2 = \quad ^\circ$

Reason:

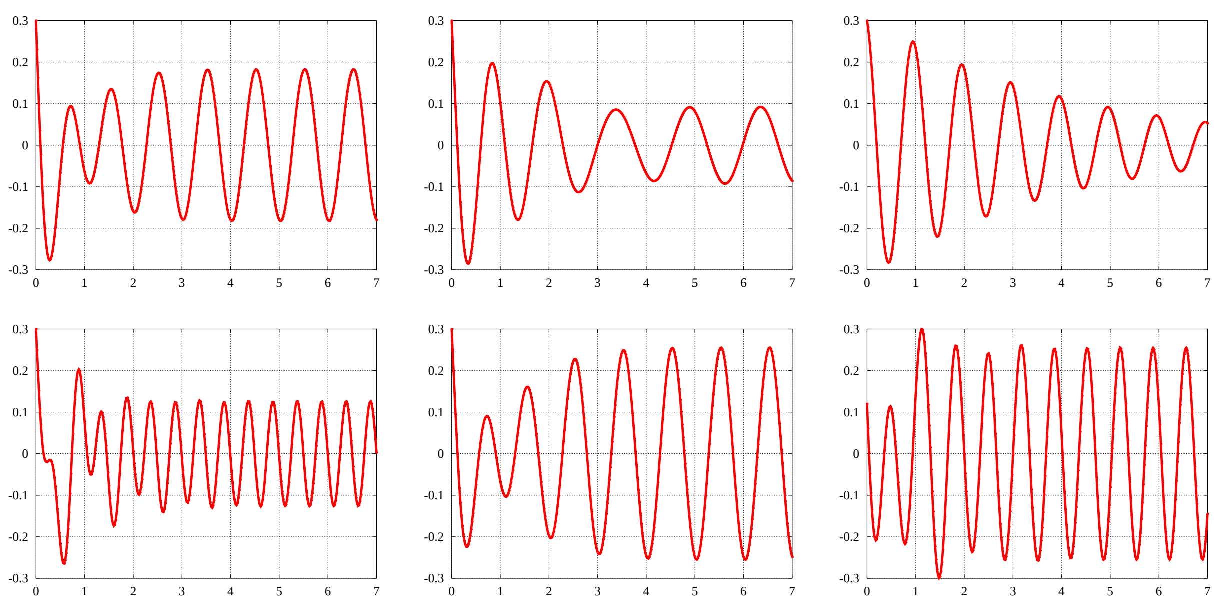
(f) **(10 pts.) Dynamic response of a building in an earthquake**

The base of a building vibrates because of an earthquake. The earth's horizontal motion is modeled as $A \sin(\Omega t)$ where A is the magnitude of the ground motion and Ω is the earthquake's frequency. The equation governing the horizontal displacement x of the building's roof is



$$\ddot{x} \;+\; 2.4\,\dot{x} \;+\; 36\,x \;=\; 0.1\,\Omega^2\,\sin(\Omega t)$$

(5 pts.) Circle the graph of $x(t)$ that corresponds to $\Omega = 1 \text{ Hz} = 6.2832$ rad/sec.
(5 pts.) Explain:



Midterm.2 (38 pts.) System response

(a) **(2 pts.)** Circle all the phrases that describe a pole at the origin of the complex plane.

Stable	**Neutrally stable**	**Unstable**	**Slow**	**Fast**
Grow	**Constant amplitude**	**Decay**	**Oscillatory**	**Damped**

(b) **(2 pts)** As a pole moves away from the imaginary axis to the left, it is more (circle all that apply)

Stable	**Neutrally stable**	**Unstable**	**Slow**	**Fast**
Grow	**Constant amplitude**	**Decay**	**Oscillatory**	**Damped**

(c) **(2 pts.)** As a pole moves away from the imaginary axis to the right, it is more

Stable	**Neutrally stable**	**Unstable**	**Slow**	**Fast**
Grow	**Constant amplitude**	**Decay**	**Oscillatory**	**Damped**

(d) **(2 pts.)** As a pole moves away (up or down) from the real axis, it is more

Stable	**Neutrally stable**	**Unstable**	**Slow**	**Fast**
Grow	**Constant amplitude**	**Decay**	**Oscillatory**	**Damped**

(e) **(2 pts.)** As a pole moves away from the origin (in any direction), it is more

Stable	**Neutrally stable**	**Unstable**	**Slow**	**Fast**
Grow	**Constant amplitude**	**Decay**	**Oscillatory**	**Damped**

(f) **(2 pts.)** A pole at $0.2 + 0.1{*}i$ is

Stable	**Neutrally stable**	**Unstable**	**Slow**	**Fast**
Grow	**Constant amplitude**	**Decay**	**Oscillatory**	**Damped**

(g) **(2 pts.)** A pole at $-2 + 30{*}i$ is

Stable	**Neutrally stable**	**Unstable**	**Slow**	**Fast**
Grow	**Constant amplitude**	**Decay**	**Oscillatory**	**Damped**

(h) **(24 pts.)** Consider the following ODEs and answer the questions about $y(t)$ with A, B, or C.

A $\ddot{y} + 6\dot{y} + 16y = 0$

B $\ddot{y} + 3.2\dot{y} + 4y = 0$

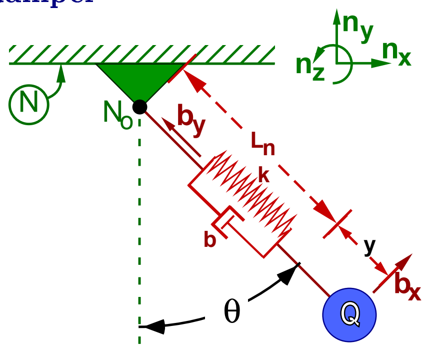
C $\ddot{y} + 1.6\dot{y} + 4y = 0$

Largest natural frequency ω_n	
Largest damping ratio ζ	
Smallest damped natural frequency ω_d	
Longest time to damp out	
Jiggles (oscillates) the fastest	
Settles the fastest	
Fastest peak time	
Largest maximum overshoot	
Largest decay ratio	
Smallest decay ratio	
Most stable	
Least stable	

Midterm.3 (36 pts.) Equations of motion for a swinging spring-damper

A straight, massless, spring-damper connects a particle Q to a point N_o which is fixed in a Newtonian reference frame N. Right-handed orthogonal unit vectors $\mathbf{n}_x$, $\mathbf{n}_y$, $\mathbf{n}_z$ are fixed in N with $\mathbf{n}_x$ directed horizontally to the right, $\mathbf{n}_y$ vertically upward, and $\mathbf{n}_z$ perpendicular to the plane in which Q moves in N. Right-handed orthogonal unit vectors $\mathbf{b}_x$, $\mathbf{b}_y$, $\mathbf{b}_z$ are fixed in a reference frame B with $\mathbf{b}_y$ directed from Q to N_o and $\mathbf{b}_z = \mathbf{n}_z$.

Other relevant identifiers are shown in the following table.



Quantity	Identifier	Type
Local gravitational constant	g	constant
Mass of Q	m	constant
Natural spring length	L_n	constant
Linear spring constant	k	constant
Linear damping constant	b	constant
Aerodynamic dynamic	b_{air}	constant
Spring stretch	y	dependent variable
Angle between $\mathbf{n}_y$ and $\mathbf{b}_y$	θ	dependent variable
Time	t	independent variable

${}^{b}R^{n}$	$\mathbf{n}_x$	$\mathbf{n}_y$	$\mathbf{n}_z$
$\mathbf{b}_x$			
$\mathbf{b}_y$			
$\mathbf{b}_z$			

(a) **(8 pts.)** Complete the previous rotation table ${}^{b}R^{n}$. Find the angular velocity and angular acceleration of B in N and express them in terms of $\mathbf{b}_x$, $\mathbf{b}_y$, $\mathbf{b}_z$.
Result:

${}^{N}\boldsymbol{\omega}^{B} =$ ${}^{N}\boldsymbol{\alpha}^{B} =$

(b) **(12 pts.)** Find the position vector of Q from N_o and the velocity and acceleration of Q in N.
Result:

$\mathbf{r}^{Q/N_o} = \quad \mathbf{b}_x + \quad \mathbf{b}_y$

${}^{N}\mathbf{v}^{Q} = \quad \mathbf{b}_x + \quad \mathbf{b}_y$

${}^{N}\mathbf{a}^{Q} = \quad \mathbf{b}_x + \quad \mathbf{b}_y$

(c) **(6 pts.)** Find $\mathbf{F}^Q$, the resultant of all *contact* and *distance* forces on Q. Model the aerodynamic damping force on Q with $-b_{air}\,({}^N\mathbf{v}^Q \cdot \mathbf{b}_x)\,\mathbf{b}_x$.

Result:

$$\mathbf{F}^Q \;=\; \qquad\qquad \mathbf{n}_y \;+\; \qquad\qquad \mathbf{b}_x \;+\; \qquad\qquad \mathbf{b}_y$$

(d) **(4 pts.)** After using Newton's law to form a vector equation of motion for Q in N, form scalar equations of motion by dot-multiplying with $\mathbf{b}_x$ and $\mathbf{b}_y$.

Result:

(e) **(4 pts.)** Classify the previous equations by picking the relevant qualifiers from the list below.

Uncoupled	**Linear**	**Homogeneous**	**Constant-coefficient**	**1st-order**	**Algebraic**
Coupled	**Nonlinear**	**Inhomogeneous**	**Variable-coefficient**	**2nd-order**	**Differential**

(f) **(2 pts.)** In a sentence, describe how the previous equations are solved to find $\theta(t)$ and $y(t)$.

Result: