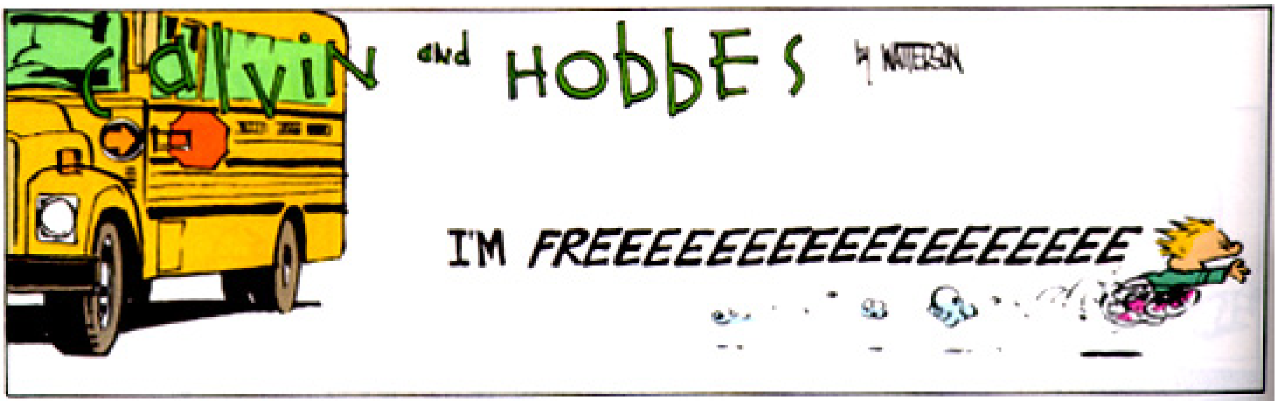
Name ______________________________

ME161 Final. Thursday December 11, 2003 7:00-10:00 p.m.

I certify that I upheld the Stanford Honor code during this exam ______________



- Print your name and sign the honor code statement
- You may use your course notes, homework, books, etc.
- Write your answers on this handout
- Where space is provided, **show your work to get credit**
- If necessary, attach extra pages for scratch work
- Best wishes for a fun vacation. Merry Christmas and Happy New Year!

Problem	Value	Score
1	2	
2	13	
3	17	
4	14	
5	10	
6	22	
7	22	
Total	100	

Final.1 (2 pts.) Class participation. Fill in the blanks.

Go ______________.

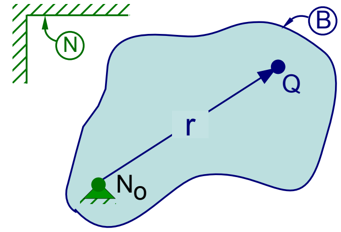
Beat __________!

Final.2 (13 pts.) Miscellaneous

(a) **(3 pts.)** The equation $\mathbf{M} = I\boldsymbol{\alpha}$ is useful for analyzing three-dimensional rotational motions of a rigid body. **True/False**

(b) **(4 pts.)**

Points N_o and Q are fixed on rigid body B. Point N_o is also fixed in reference frame N. Show that ${}^N\mathbf{v}^Q$ (the velocity of Q in N) can be written in terms of ${}^N\boldsymbol{\omega}^B$ (the angular velocity of B in N) and $\mathbf{r}$ (the position vector from N_o to Q). Explain each step in your mathematical proof with a brief phrase.

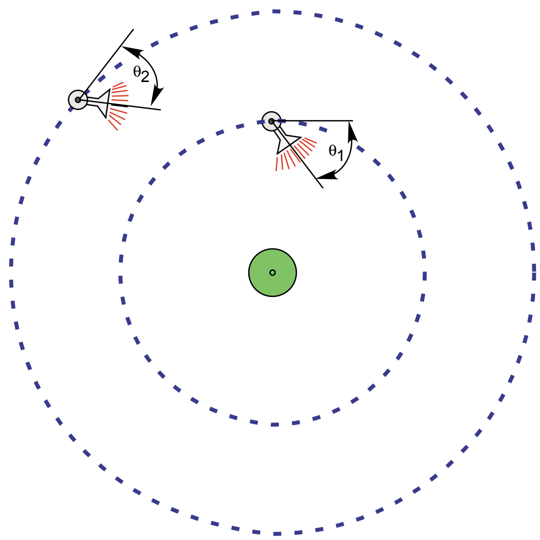


Mathematical statement	Reasoning
${}^N\mathbf{v}^Q \triangleq$	Definition of velocity of Q in N

(c) **(4 pts.) Minimum fuel-use orbit transfer**
To thrust a satellite from low circular orbit about Earth to a higher circular orbit, an impulse is provided at two instants. The first impulse can be directed radially outward, tangent to the satellite's circular orbit, or directed at some angle θ_1 from the satellite's orbital tangent. The second impulse is applied at apogee (when the satellite is furthest from Earth) and is directed at an angle θ_2 from the orbital tangent.

The first impulse puts the satellite into an elliptical orbit and the second changes the orbit from elliptical to circular.

Using your engineering insights, provide values for θ_1 and θ_2 that minimize the amount of fuel required for this orbital transfer. Explain your reason for choosing these values.



Result:

$\theta_1 =$ $\quad^\circ$ $\qquad\qquad$ $\theta_2 =$ $\quad^\circ$

Reason:

(d) **(2 pts.)** Put the following functions into amplitude/phase form by filling in the phase.[40]

$$-\sin(2t) = \sin(2t + \quad)$$
$$-\cos(2t) = \sin(2t + \quad)$$

[40]In amplitude/phase form, the amplitude is non-negative.

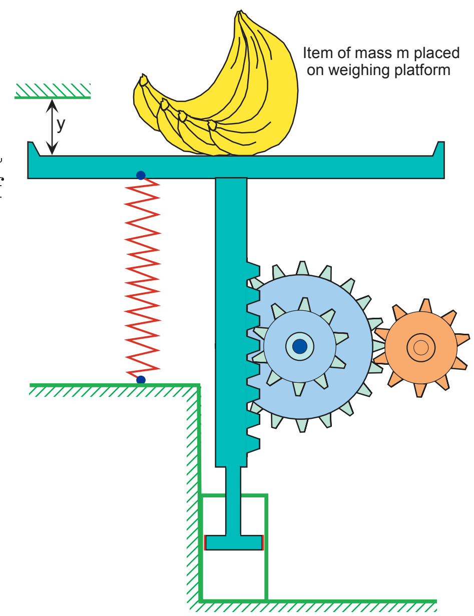
Final.3 (17 pts.) Finding mass, damping, and spring constants to meet design specifications

One task an engineer performs is sizing springs, dampers, etc., to meet design specifications. Knowing that the equation governing the deflection $y(t)$ for the spring scale shown to the right is

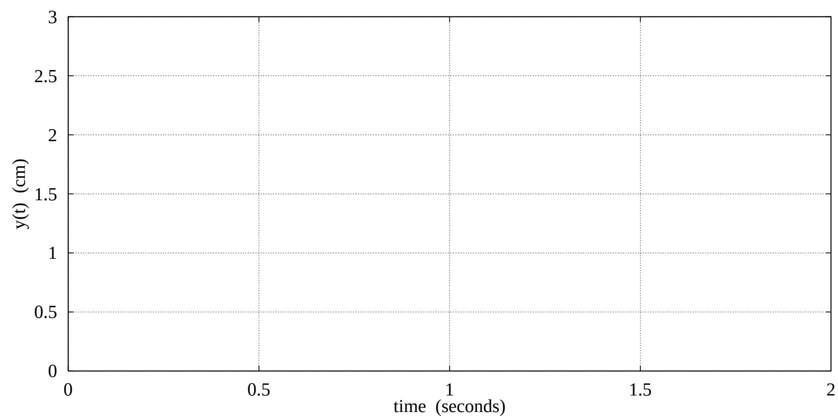
$$(m+m_e)\,\ddot{y} \;+\; b_e\,\dot{y} \;+\; k_e\,y \;=\; 9.8\,m$$

determine numerical values for m_e, b_e, and k_e so that

- the scale deflects y=2 cm when m=10 kg
- the scale settles to 1% of its final value within 2 seconds
- the maximum overshoot of $y(t)$ from its final value is 50%



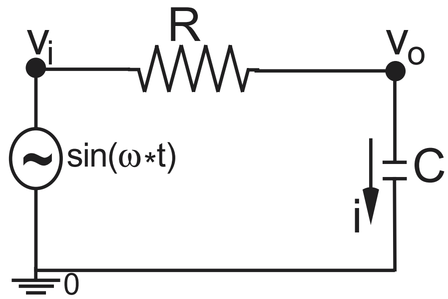
To ensure you understand the design specifications, first make a *rough* sketch of $y(t)$ for $0 \le t \le 2$ sec and clearly identify each of the design specifications on the graph. Assume $y(0)$=0 and $\dot{y}(0)$=0. **(5 pts.)**



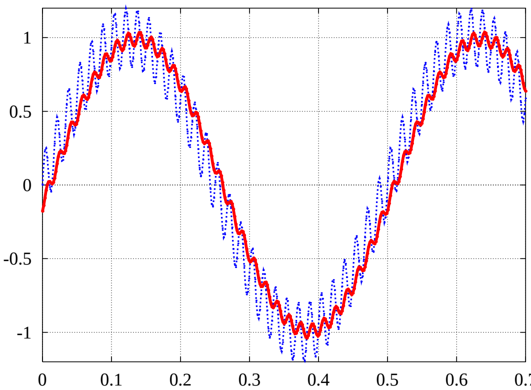
(12 pts.) Result:

m_e	b_e	k_e
kg	$\frac{\text{n*sec}}{\text{m}}$	$\frac{\text{n}}{\text{m}}$

Final.4 (10 pts.) Design of a low-pass filter



The electrical circuit to the right is an RC (resistor-capacitor) circuit that acts as a filter. The input voltage signal v_i is a known (specified) function of time t, and the output quantity of interest is $v_o(t)$, the voltage across the capacitor.



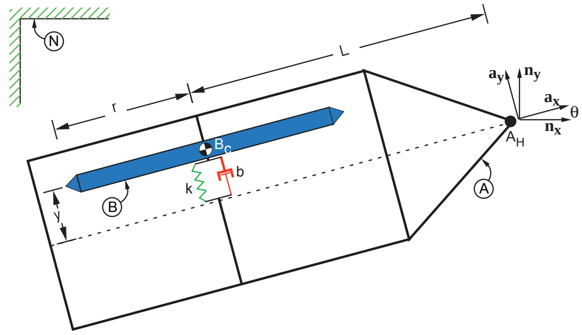
The input signal v_i comes from a sensor that is "noisy" because it transmits ambient 60 Hz frequencies which need to be filtered out. In other words, the sensor transmits a signal such as the thin dotted-line shown to the right.
Knowing that C=1milliFarad, find a value for R so that the filter's bandwidth is 15 Hz and the output signal $v_o(t)$ looks like the thick solid-line shown to the right

Result:

$R =$ Ohms

Final.5 (22 pts.) Linearization, state-space, and stability of a single-wheel trailer

Although it is well know that single-wheel trailers sometimes behave poorly, it is not always clear why they do so. One possibility is that tire flexibility and loose wheel mounting give rise to unstable behavior. To explore this concept, the following identifiers are useful.



Quantity	Identifier	Type
Distance from A_H (hitch point) to the wheel's axle	L	constant
Wheel radius	r	constant
Linear spring constant modeling tire flexibility	k	constant
Linear damping constant modeling tire flexibility	b	constant
Mass of B	m	constant
Moment of inertia of A about A_H for $\mathbf{a}_z$	I^A	constant
Moment of inertia of B about B_c for $\mathbf{a}_y$	I^B	constant
Moment of inertia of B about B_c for *any* line perpendicular to $\mathbf{a}_y$	J^B	constant
$\mathbf{n}_x$ measure of the velocity of A_H in N	v	constant
Angle between $\mathbf{n}_x$ and $\mathbf{a}_x$	θ	dependent variable
Distance between B_c and the line parallel to $\mathbf{a}_x$ that passes through A_H	y	dependent variable
Time	t	independent variable

The equations governing this system's motion can be written in terms of $z_1 \triangleq m + \frac{I^B}{r^2}$ as

$$\dot{y} \;-\; v\,\sin(\theta) \;-\; L\,\dot{\theta} \;=\; 0$$

$$(I^A + J^B + z_1\,y^2)\,\ddot{\theta} \;+\; L\,z_1\,y\,\dot{\theta}^2 \;+\; [b\,L^2 + 2\,V\,z_1\,y\,\sin(\theta)]\,\dot{\theta} \;+\; L\,[k\,y + b\,V\,\sin(\theta)] \;=\; 0$$

(a) **(4 pts.)** Assuming y, θ, and $\dot{\theta}$ are small, linearize the nonlinear differential equations.
Result:

$$= \; 0$$

$$= \; 0$$

(b) **(3 pts.)** After solving the last equation for $\ddot{\theta}$, cast the previous set of equations into the state-space form $\dot{Y} = A\,Y$ where $Y \triangleq \begin{bmatrix} y \\ \theta \\ \dot{\theta} \end{bmatrix}$, and find each element of A.

Result:

$$\begin{bmatrix} \dot{y} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix} \begin{bmatrix} y \\ \theta \\ \dot{\theta} \end{bmatrix}$$

(c) **(12 pts.)** The solution to the previous set of equations has the form $Y(t) = U * e^{\lambda t}$ where λ is a constant and U is a *non-zero* 3×1 matrix of constants. Find the polynomial equation which governs the value of λ.
Result:

$$\lambda^3 + \underline{\hspace{3cm}} * \lambda^2 + \underline{\hspace{3cm}} * \lambda + \underline{\hspace{3cm}} = 0$$

(d) **(3 pts.)** For a certain trailer, the polynomial equation governing λ is

$$\lambda^3 + 5.06\,\lambda^2 + (50.6 + 3.4 * v)\,\lambda + 33.7 * v = 0$$

How does one determine the values of v which result in an unstable trailer motion?
Result: