Math 51 - Winter 2011 - Midterm Exam II

Name: ____________________________________

Student ID: ________________________________

Circle your section meting time:

Nick Haber 11:00 AM 1:15 PM	James Zhao 10:00 AM 1:15 PM	Henry Adams 11:00 AM 1:15 PM
Ralph Furmaniak 11:00 AM 1:15 PM	Jeremy Miller 11:00 AM 2:15 PM	Ha Pham 11:00 AM 1:15 PM
Sukhada Fadnavis 10:00 AM 1:15 PM	Max Murphy 11:00 AM 1:15 PM	Jesse Gell-Redman 1:15 PM

Signature: ________________________________

Instructions: Print your name and student ID number, select the time at which your section meets, and **write your signature to indicate that you accept the Honor Code**. There are 10 problems on the pages numbered from 1 to 13. Each problem is worth 10 points. In problems with multiple parts, the parts are worth an equal number of points unless otherwise noted. Please check that the version of the exam you have is complete, and correctly stapled. In order to receive full credit, please show all of your work and justify your answers. You may use any result from class, but if you cite a theorem be sure to verify the hypotheses are satisfied. **You have 1 hour and 30 minutes.** This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted. GOOD LUCK!

Question	1	2	3	4	5	6	7	8	9	10	Total
Score											

1(a). Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 1 \end{bmatrix}$.

Solution: To find the inverse of a matrix A, we reduce the augmented matrix $[A|Id]$ to $[Id|A^{-1}]$.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 1 & 0 \\ 2 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & -2 & -2 & -2 & 1 & 0 \\ 0 & -2 & -3 & -2 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & -2 & -2 & -2 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & -2 & 0 & -2 & 3 & -2 \\ 0 & 0 & -1 & 0 & -1 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 & -\frac{3}{2} & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array}\right]$$

Also, notice that the inverse of a symmetric matrix is symmetric.

1(b). Find all x for which the matrix

$$\begin{bmatrix} x-2 & 5 & 1 \\ -1 & 0 & x \\ -2 & 1 & 2 \end{bmatrix}$$

is **not** invertible.

Solution: The matrix is non-invertible whenever its determinant is zero. By applying expansion by minors along the central column, we get

$$\begin{aligned} \begin{vmatrix} x-2 & 5 & 1 \\ -1 & 0 & x \\ -2 & 1 & 2 \end{vmatrix} &= -5 \begin{vmatrix} -1 & x \\ -2 & 2 \end{vmatrix} - \begin{vmatrix} x-2 & 1 \\ -1 & x \end{vmatrix} \\ &= -5(-2+2x) - (x(x-2)+1) \\ &= -x^2 - 8x + 9 \\ &= -(x-9)(x+1) \end{aligned}$$

Which equals zero when x is either 1 or -9.

2. Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation defined by:

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+y \\ y \end{bmatrix}.$$

(a). Find the matrix A that represents the linear transformation T with respect to the standard basis $\mathcal{S} = \{\mathbf{e}_1, \mathbf{e}_2\}$.

Solution: In general, the matrix A that represents the linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ has as first column

$$T\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and as second column

$$T\begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

In this case,

$$T\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \; T\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

so

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

Grading: This part is out of 3 points. Not much partial credit was given.

(b). Consider the basis $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$ given by: $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Find the change of basis matrix C for the basis $\mathcal{B}$. That is, find the matrix C such that $\mathbf{v} = C[\mathbf{v}]_{\mathcal{B}}$ for all vectors $\mathbf{v}$.

Solution: In general, if we have a basis $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$ for $\mathbf{R}^2$, then if we take C to have first column $\mathbf{v}_1$ and second column $\mathbf{v}_2$, then $\mathbf{v} = C[\mathbf{v}]_{\mathcal{B}}$ for all vectors $\mathbf{v}$. Thus in this case,

$$C = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}.$$

Grading: This part is out of 2 points. All or nothing.

(c). Find the matrix B that represents the linear transformation T with respect to the basis $\mathcal{B}$.

Solution: In general, $B = C^{-1}AC$. We have

$$C^{-1} = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}.$$

This can be found either via the standard augmented matrix method for finding the inverse, or by remembering the 2×2 case: if $C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $C^{-1} = \frac{1}{\det C}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. To find the final answer, we multiply

$$\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}.$$

Remark: An alternate approach would be to note that B is the matrix that has as first column $[T(\mathbf{v}_1)]_{\mathcal{B}}$, and as second column $[T(\mathbf{v}_2)]_{\mathcal{B}}$. We have

$$T\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} = 2\begin{bmatrix} 3 \\ 1 \end{bmatrix} - 1\begin{bmatrix} 2 \\ 1 \end{bmatrix},$$

and

$$T\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} = 1\begin{bmatrix} 3 \\ 1 \end{bmatrix} + 0\begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

A common mistake was to leave these in the standard basis representation, i.e. give as answer

$$\begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix},$$

which is wrong.

Grading: This part is out of 5 points. Computing CAC^{-1} instead of $C^{-1}AC$ is worth at most 2/5, as this is a rather conceptual error.

3(a). Find all eigenvalues of the matrix $A = \begin{bmatrix} -1 & 2 & 2 \\ 2 & 2 & -1 \\ 2 & -1 & 2 \end{bmatrix}$.

Solution: The characteristic polynomial is

$$P_A(\lambda) = \det(\lambda I - A) = \begin{vmatrix} \lambda+1 & -2 & -2 \\ -2 & \lambda-2 & 1 \\ -2 & 1 & \lambda-2 \end{vmatrix}$$

$$= (\lambda+1)\begin{vmatrix} \lambda-2 & 1 \\ 1 & \lambda-2 \end{vmatrix} - (-2)\begin{vmatrix} -2 & 1 \\ -2 & \lambda-2 \end{vmatrix} + (-2)\begin{vmatrix} -2 & \lambda-2 \\ -2 & 1 \end{vmatrix}$$

$$= (\lambda+1)(\lambda^2 - 4\lambda + 4 - 1) + 2(-2(\lambda-2)+2) - 2(-2+2(\lambda-2))$$

$$= \lambda^3 - 3\lambda^2 - \lambda + 3 - 4\lambda + 8 + 4 + 4 - 4\lambda + 8$$

$$= \lambda^3 - 3\lambda^2 - 9\lambda + 27.$$

Factorising this requires a little trial and error. All the coefficients are powers of 3, so one might try $\lambda = 3$. Since $P_A(3) = 0$, this confirms that $\lambda - 3$ is a factor. Hence,

$$P_A(\lambda) = (\lambda-3)(\lambda^2-9) = (\lambda-3)(\lambda-3)(\lambda+3).$$

Thus, the eigenvalues are 3 (with multiplicity 2) and -3.

Remark: The second and third terms in the determinant calculation do NOT cancel. Consequential credit was not awarded for this mistake as it made the question significantly easier.

3(b). Consider the matrix $B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$.

Find an eigenvector of B with eigenvalue $\lambda = 1$.

Solution: The eigenspace is

$$E_1 = N(I - B) = N\left(\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}\right).$$

This corresponds to the equations $x_2 = 0$ and $2x_3 = 0$, while x_1 is a free variable, hence the general eigenvector is $(x_1, 0, 0)$. We can pick any of them for the answer, for example, $(1, 0, 0)$.

Remarks: Students who found $RREF(I - B)$ but wrote down an incorrect eigenvector were awarded 2/5—while the matrix was potentially confusing due to its zero column, it would only have taken 10 seconds to mentally check the answer! Students who gave the zero vector as an answer were awarded 0/5, as it shows both a misunderstanding of RREF and a misunderstanding of eigenvectors.

4(a). Find the eigenvalues of the matrix $A = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

Solution: Let

$$A = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

To calculate the eigenvalues of A we form

$$\lambda I_3 - A = \begin{bmatrix} \lambda - 4 & -2 & 0 \\ -2 & \lambda - 1 & 0 \\ 0 & 0 & \lambda - 2 \end{bmatrix}$$

and we need to solve

$$\begin{aligned} 0 &= det(\lambda I_3 - A) \\ &= (\lambda - 2)\left[(\lambda - 4)(\lambda - 1) - 4\right] - 4(\lambda - 2) \\ &= (\lambda - 2)\lambda(\lambda - 5). \end{aligned}$$

So the eigenvalues of A are $\lambda = 0, 2, 5$.

4(b). Consider the quadratic form $(A\mathbf{x}) \cdot \mathbf{x}$ (or in the other notation $\mathbf{x}^T A\mathbf{x}$), where A is the matrix in part (a).

Determine whether the quadratic form is positive definite, indefinite, or negative definite. If it is none of those, determine whether the quadratic form is positive semidefinite or negative semidefinite.

Solution: The quadratic form associated with A is positive semidefinite. This follows since the eigenalues of A are all nonnegative and there is a zero eigenvalue.

Common Mistakes:

In a) many people thought that since A is symmetric that the eigenvalues of A are just the diagonal elements of A. This is wrong. This is true when A is upper triangular or lower triangular.
In b) many people used the result that relates the determinant of A and the trace of A to what definiteness the quadratic form associated to A is. This result is only valid in the 2×2 case. Further more there is really no math needed for part b) since you already know the eigenvalues of A from part a).

5.The position of a particle at time t is $\mathbf{u}(t) = (\sin t, t^2, \cos t)$.

(a). Find the velocity of the particle at time t.

Solution:

$$\mathbf{v}(t) = (\cos(t), 2t, -\sin(t)).$$

(b). Find the acceleration of the particle at time t.

Solution:

$$\mathbf{a}(t) = (-\sin(t), 2, -\cos(t)).$$

(c). Find the speed of the particle at time t.

Solution:

$$||\mathbf{v}(t)|| = \sqrt{\cos^2(t) + 4t^2 + (-\sin(t))^2} = \sqrt{1 + 4t^2}.$$

(d). Find the tangent line to the path of the particle at the point $(0, 0, 0)$.

Solution: $\mathbf{u}(t) = (0, 0, 1)$ when $t = 0$. Equation of tangent at $(0, 0, 1)$ is thus,

$$\{\mathbf{u}(0) + s\mathbf{v}(0) \mid s \in \mathbf{R}\} = \{(0, 0, 1)^T + s(1, 0, 0)^T \mid s \in \mathbf{R}\}$$

6.Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the reflection across the line $y = -x$.

(a). Find the matrix for T (with respect to the standard basis of $\mathbf{R}^2$.)

Solution: We denote by $(\mathbf{e}_1, \mathbf{e}_2)$ the standard basis of $\mathbf{R}^2$.

$$T(\mathbf{e}_1) = -\mathbf{e}_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}.$$

$$T(\mathbf{e}_2) = -\mathbf{e}_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}.$$

The matrix for T in the standard basis of $\mathbf{R}^2$ is thus:

$$A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}.$$

Remark: Many students wrote the matrix for T in the basis $\{\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \end{bmatrix}\}$. If you do this, you have to make a change of basis to get the correct result (**in the standard basis of $\mathbf{R}^2$.**)

(b). Let $R : \mathbf{R}^2 \to \mathbf{R}^2$ be the rotation with angle π, and T the same as in $6(a)$. Find the matrix for $T \circ R$ (with respect to the standard basis of $\mathbf{R}^2$.)

Solution: Matrix for a rotation of angle θ: $B = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$.
Here $\cos(\pi) = -1$ and $\sin(\pi) = 0$. So the matrix for a rotation of angle π is:

$$B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

The linear transformation $T \circ R$ has matrix:

$$AB = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

7. The temperature at a point x at time t on a heated wire is given by

$$f(x,t) = \sin((tx)^2 - 34)$$

7(a). Compute both of the partial derivatives of f.

Solution: Using the chain rule to differentiate $f(x,t) = \sin((tx)^2 - 34)$ we get

$$\frac{\partial f}{\partial x} = \cos((tx)^2 - 34)2xt^2$$

and

$$\frac{\partial f}{\partial t} = \cos((tx)^2 - 34)2tx^2.$$

7(b). Is the temperature at the point $x = 2$ decreasing or increasing at time $t = 2$?

Solution: For a fixed $x = 2$ the change in temperature is given by $\frac{\partial f}{\partial t}$. So at $t = 2$ we need to see whether

$$\frac{\partial f}{\partial t}(2,2) = \cos((4)^2 - 34) \cdot 2 \cdot 2 \cdot 2^2 = \cos(-18)16$$

is positive or negative. Now since $\pi \approx 3.14$ we have

$$-5\frac{1}{2}\pi > -18 > -6\pi$$

which implies $\cos(-18) > 0$. Thus $\frac{\partial f}{\partial t}(2,2) > 0$ and so the temperature is increasing.

8. Suppose $F : \mathbf{R}^3 \to \mathbf{R}^2$ is defined by

$$F(x, y, z) = \begin{bmatrix} \sin(x \cos y) \\ x + 2y + \sin x \end{bmatrix}.$$

Find the Jacobian matrix (i.e, the matrix for the total derivative) $D_F(0, 0, 1)$.

Solution: If $F : \mathbf{R}^3 \to \mathbf{R}^2$ is given by

$$F(x, y, z) = \begin{bmatrix} \sin(x \cos y) \\ x + 2y + \sin x \end{bmatrix} = \begin{bmatrix} f(x, y) \\ g(x, y) \end{bmatrix}$$

then

$$D_F = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} & \frac{\partial g}{\partial z} \end{bmatrix}$$

$$\begin{aligned}
\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x} \sin(x \cos y) = (\cos y) \cos(x \cos y) && \Rightarrow \frac{\partial f}{\partial x}(0, 0, 1) = 1 \\
\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y} \sin(x \cos y) = -x(\sin y) \cos(x \cos y) && \Rightarrow \frac{\partial f}{\partial y}(0, 0, 1) = 0 \\
\frac{\partial g}{\partial x} &= \frac{\partial}{\partial x} x + 2y + \sin x = 1 + \cos x && \Rightarrow \frac{\partial g}{\partial x}(0, 0, 1) = 2 \\
\frac{\partial g}{\partial y} &= \frac{\partial}{\partial y} x + 2y + \sin x = 2 && \Rightarrow \frac{\partial g}{\partial y}(0, 0, 1) = 2 \\
\frac{\partial f}{\partial z} &= \frac{\partial f}{\partial z} = 0
\end{aligned}$$

Hence

$$D_F(0, 0, 1) = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \end{bmatrix}$$

Common mistakes

- Students leave out the column of differentiating f and g with respect to z so the matrix ends up being only 2×2 instead of 2×3. (*In this case student will have at most 6pts if the calculations for other entries are correct*)
- Students write down D_F^t instead of D_F. (*In this case, students receive at most 8pts if the calculations for other entries are correct*)
- The remaining error are either not knowing $\cos(0) = 1$ or not knowing how to differentiate $\sin(x \cos y)$. A lot of students get $\frac{\partial}{\partial y} \sin(x \cos y)$ wrong.

9. Let
$$f(x,y) = \frac{x^2y + xy^2 + y^3}{x^2 + y^2}.$$

9(a). Find
$$\lim_{(x,y)\to(0,0)} f(x,y)$$
if the limit exists.

Solution: Let $r = \sqrt{x^2 + y^2}$. Then $|x| \le r$ and $|y| \le r$. Thus by the triangle inequality,
$$|x^2y + xy^2 + y^3| \le |x^2y| + |y^2x| + |y^3| \le 3r^3.$$

Consequently, we have the inequalities
$$-3r \le \frac{x^2y + xy^2 + y^3}{x^2 + y^2} \le 3r.$$

Now,
$$\lim_{(x,y)\to(0,0)} \sqrt{x^2 + y^2} = 0$$
so at this point in time, we've bounded $f(x,y)$ from above and below by functions that have limit 0 as (x,y) tends to $(0,0)$. Thus (by the Squeeze theorem)
$$\lim_{(x,y)\to(0,0)} f(x,y) = 0.$$

Commentary: It is possible to do this problem entirely in the language of polar coordinates, which in fact was the choice of most successful students. Note that in order to show that the limit exists, it is insufficient to compute the limit along all lines passing through $(0,0)$.

9(b). Compute $\frac{\partial f}{\partial x}$, and use this to determine

$$\lim_{(x,y)\to(0,0)} \frac{\partial f}{\partial x}$$

if the limit exists.

Solution: First we need to compute $\frac{\partial f}{\partial x}$ (recall the quotient rule).

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{(2xy+y^2)(x^2+y^2)-(2x)(x^2y+xy^2+y^3)}{(x^2+y^2)^2} \\ &= \frac{y^4-x^2y^2}{(x^2+y^2)^2}.\end{aligned}$$

We shall show that $\lim_{(x,y)\to(0,0)} \frac{\partial f}{\partial x}$ does not exist by calculating the limits obtained by approaching $(0,0)$ along two different curves.

First, let us approach along the x-axis (so $y = 0$). We calculate $\frac{\partial f}{\partial x}(x,0) = 0$, which clearly has limit 0. Now let us approach along the y-axis (so $x = 0$). We calculate $\frac{\partial f}{\partial x}(0,y) = 1$, which clearly has limit 1. Note that $1 \neq 0$, so the limit asked for in the question cannot exist.

Commentary: A common pitfall in the algebraic manipulations required here is to lose track of the exponent. A useful sanity check in this case is to notice for example that all terms in the numerator of the derivative have to have degree 4 (it is homogenous). This guards against some set of possible errors (and those sorts of errors unfortunately, have the tendency to make the limit calculation much easier, invalidating the possibility of obtaining 'follow through' marks).

10. Let $g : \mathbf{R}^2 \longrightarrow \mathbf{R}^2$ be the function

$$g(x, y) = (\sin(x + 3y), xy^2 + y)$$

and suppose that f is a function defined on a neighborhood of $(0, 0)$, such that the composition $f \circ g$ is the identity function. Find $D_f(0, 0)$.

Solution: Here the "neighborhood hypothesis", is to guarantee that both the composition $f \circ g$ and the derivative make sense. Alright, since $f \circ g$ is the identity function, that means that $f \circ g(x, y) = (x, y)$. Computing the Jacobian matrix and then using the chain rule we get:

$$\begin{aligned} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} &= D_{f \circ g}(0,0) = D_f(g(0,0)) \cdot D_g(0,0) \\ &= D_f(0,0) \cdot D_g(0,0) \end{aligned}$$

Now, since

$$D_g(x,y) = \begin{bmatrix} \frac{\partial}{\partial x}\sin(x+3y) & \frac{\partial}{\partial y}\sin(x+3y) \\ \frac{\partial}{\partial x}(xy^2+y) & \frac{\partial}{\partial y}(xy^2+y) \end{bmatrix} = \begin{bmatrix} \cos(x+3y) & 3\cos(x+3y) \\ y^2 & 2xy+1 \end{bmatrix}$$

we get

$$D_g(0,0) = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

Notice that the matrix $D_g(0, 0)$ is invertible, and since

$$I_2 = D_f(0,0) \cdot D_g(0,0)$$

we conclude that $D_f(0, 0) = D_g(0, 0)^{-1}$. Computing the inverse, we obtain

$$D_f(0,0) = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$$

and we are done.

Commentary: Maybe the most common mistake in this problem was to claim that f and g must be inverses. This is not necessarily true, and 2 points were deducted because of this. The definition of inverses says that both $f \circ g$ and $g \circ f$ are the identity function. Take for example $g : \mathbf{R} \longrightarrow \mathbf{R}$ to be $g(x) = x$ and let

$$f(x) = \begin{cases} x, & \text{if } |x| < 1 \\ 0, & \text{if } |x| \geq 1 \end{cases}$$

so $f \circ g(x) = x$ in a neighborhood of zero (which is the only information given in the problem), but f and g are not inverses. Other errors were to claim that the derivative of the identity was zero, and not to use/misuse the chain rule (-2 points) .