Math 51 - Winter 2011 - Midterm Exam II

Name: ______________________________

Student ID: ______________________________

Circle your section meting time:

Nick Haber 11:00 AM 1:15 PM	James Zhao 10:00 AM 1:15 PM	Henry Adams 11:00 AM 1:15 PM
Ralph Furmaniak 11:00 AM 1:15 PM	Jeremy Miller 11:00 AM 2:15 PM	Ha Pham 11:00 AM 1:15 PM
Sukhada Fadnavis 10:00 AM 1:15 PM	Max Murphy 11:00 AM 1:15 PM	Jesse Gell-Redman 1:15 PM

Signature: ______________________________

Instructions: Print your name and student ID number, select the time at which your section meets, and **write your signature to indicate that you accept the Honor Code**. There are 10 problems on the pages numbered from 1 to 13. Each problem is worth 10 points. In problems with multiple parts, the parts are worth an equal number of points unless otherwise noted. Please check that the version of the exam you have is complete, and correctly stapled. In order to receive full credit, please show all of your work and justify your answers. You may use any result from class, but if you cite a theorem be sure to verify the hypotheses are satisfied. **You have 1 hour and 30 minutes.** This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted. GOOD LUCK!

Question	1	2	3	4	5	6	7	8	9	10	Total
Score											

1(a). Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 1 \end{bmatrix}$.

1(b). Find all x for which the matrix

$$\begin{bmatrix} x-2 & 5 & 1 \\ -1 & 0 & x \\ -2 & 1 & 2 \end{bmatrix}$$

is **not** invertible.

2. Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation defined by:

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+y \\ y \end{bmatrix}.$$

(a). Find the matrix A that represents the linear transformation T with respect to the standard basis $\mathcal{S} = \{\mathbf{e}_1, \mathbf{e}_2\}$.

(b). Consider the basis $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$ given by: $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Find the change of basis matrix C for the basis $\mathcal{B}$. That is, find the matrix C such that $\mathbf{v} = C[\mathbf{v}]_{\mathcal{B}}$ for all vectors $\mathbf{v}$.

(c). Find the matrix B that represents the linear transformation T with respect to the basis $\mathcal{B}$.

3(a). Find all eigenvalues of the matrix $A = \begin{bmatrix} -1 & 2 & 2 \\ 2 & 2 & -1 \\ 2 & -1 & 2 \end{bmatrix}$.

3(b). Consider the matrix $B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$.

Find an eigenvector of B with eigenvalue $\lambda = 1$.

4(a). Find the eigenvalues of the matrix $A = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

4(b). Consider the quadratic form $(A\mathbf{x}) \cdot \mathbf{x}$ (or in the other notation $\mathbf{x}^T A\mathbf{x}$), where A is the matrix in part (a).

Determine whether the quadratic form is positive definite, indefinite, or negative definite. If it is none of those, determine whether the quadratic form is positive semidefinite or negative semidefinite.

5.The position of a particle at time t is $\mathbf{u}(t) = (\sin t, t^2, \cos t)$.

(a). Find the velocity of the particle at time t.

(b). Find the acceleration of the particle at time t.

(c). Find the speed of the particle at time t.

(d). Find the tangent line to the path of the particle at the point $(0, 0, 0)$.

6.Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the reflection across the line $y = -x$.

(a). Find the matrix for T (with respect to the standard basis of $\mathbf{R}^2$.)

(b). Let $R : \mathbf{R}^2 \to \mathbf{R}^2$ be the rotation with angle π, and T the same as in $6(a)$. Find the matrix for $T \circ R$ (with respect to the standard basis of $\mathbf{R}^2$.)

7. The temperature at a point x at time t on a heated wire is given by

$$f(x,t) = \sin((tx)^2 - 34)$$

7(a). Compute both of the partial derivatives of f.

7(b). Is the temperature at the point $x = 2$ decreasing or increasing at time $t = 2$?

8. Suppose $F : \mathbf{R}^3 \to \mathbf{R}^2$ is defined by

$$F(x, y, z) = \begin{bmatrix} \sin(x \cos y) \\ x + 2y + \sin x \end{bmatrix}.$$

Find the Jacobian matrix (i.e, the matrix for the total derivative) $D_F(0, 0, 1)$.

9. Let

$$f(x,y) = \frac{x^2y + xy^2 + y^3}{x^2 + y^2}.$$

9(a). Find

$$\lim_{(x,y)\to(0,0)} f(x,y)$$

if the limit exists.

9(b). Compute $\frac{\partial f}{\partial x}$, and use this to determine

$$\lim_{(x,y)\to(0,0)} \frac{\partial f}{\partial x}$$

if the limit exists.

10. Let $g : \mathbf{R}^2 \longrightarrow \mathbf{R}^2$ be the function

$$g(x, y) = (\sin(x + 3y), xy^2 + y)$$

and suppose that f is a function defined on a neighborhood of $(0, 0)$, such that the composition $f \circ g$ is the identity function. Find $D_f(0, 0)$.