MATH 51 MIDTERM 2 (MARCH 4, 2010)

Max Murphy 11am 1:15pm	Jonathan Campbell 11am 2:15pm	Jon Lee 10am 1:15pm	Eric Malm 11am 1:15pm
Xin Zhou 11am 1:15pm	Ken Chan (ACE) 1:15pm	Jose Perea 11am 1:15pm	Frederick Fong 11am 1:15pm

Your name (print):

Sign to indicate that you accept the honor code:

Instructions: Find your TA's name in the table above, and circle the time that your TTh section meets. During the test, you may not use notes, books, or calculators. Read each question carefully, and show all your work. Each of the 10 problems is worth 10 points. You have 90 minutes to do all the problems.

1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
Total	

1(a). Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & -2 \\ 0 & 2 & -2 \\ -1 & -1 & 3 \end{bmatrix}$.

1(b). Find all x for which the matrix

$$\begin{bmatrix} 3 & 4 & x \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

is *not* invertible.

2. Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation defined by:

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+y \\ -2x+4y \end{bmatrix}.$$

(a). Find the matrix A that represents the linear transformation T with respect to the standard basis $\mathcal{S} = \{\mathbf{e}_1, \mathbf{e}_2\}$.

(b). Consider the basis $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$ given by: $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$. Find the change of basis matrix C for the basis $\mathcal{B}$. That is, find the matrix C such that $\mathbf{v} = C[\mathbf{v}]_{\mathcal{B}}$ for all vectors $\mathbf{v}$.

(c). Find the matrix B that represents the linear transformation T with respect to the basis $\mathcal{B}$.

3(a). Find all eigenvalues of the matrix $A = \begin{bmatrix} 1 & 5 & 2 \\ 0 & 7 & 0 \\ 1 & 3 & 3 \end{bmatrix}$.

3(b). Consider the matrix $B = \begin{bmatrix} 7 & 5 & -7 \\ -5 & -3 & 6 \\ 1 & 1 & 0 \end{bmatrix}$.

Find an eigenvector of B with eigenvalue $\lambda = 1$.

4(a). Find the eigenvalues of the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

4(b). Consider the quadratic form $\mathbf{x}^T A\mathbf{x}$, where A is the matrix in part (a).

Determine whether the quadratic form is positive definite, indefinite, or negative definite. If it is none of those, determine whether the quadratic form is positive semidefinite or negative semidefinite.

5(a). Consider a symmetric matrix B such that

$$B\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix} \quad \text{and} \quad B\begin{bmatrix}5\\7\\0\end{bmatrix} = 3\begin{bmatrix}5\\7\\0\end{bmatrix}.$$

Find a basis of $\mathbf{R}^3$ consisting of eigenvectors of B.

5(b). If the determinant of B is 7, what are its eigenvalues? (Here B is the matrix from part (a).)

6(a).The position of a particle at time t is $\mathbf{u}(t) = (t, t^2, t^3)$. Find the velocity of the particle at time t.

6(b). Find the acceleration of the particle at time t.

6(c). Find the speed of the particle at time t.

6(d). Find the tangent line to the path of the particle at the point $(1, 1, 1)$.

7. The temperature at point (x, y) on the floor of a room is given by $f(x, y) = xy^2$.

7(a). A tweetle beetle crawls on the floor. At time $t = 2$, he is at the point $(1, 3)$ and his velocity is $(2, -1)$. Let $u(t)$ be the temperature where the beetle is at time t. Find $u'(2)$.

7(b). Another tweetle beetle is at the point $(1, 3)$, where she finds it uncomfortably cold. In which direction should she start moving to warm up as quickly as possible?

8. Suppose $F : \mathbf{R}^3 \to \mathbf{R}^2$ is defined by

$$F(x, y, z) = \begin{bmatrix} xy \\ x + 2y + \sin z \end{bmatrix}.$$

Find the Jacobian matrix (i.e, the matrix for the total derivative) $DF(1, 2, 0)$.

9. In part (a) and (b), find the indicated limit or else show that the limit does not exist.

9(a). $\lim_{(x,y)\to(0,0)} \dfrac{xy}{x^2+2y^2}$.

9(b). $\lim_{(x,y)\to(0,0)} \dfrac{xy^2}{x^2+y^2}$.

10. Suppose $f : \mathbf{R}^2 \to \mathbf{R}^2$ is defined by $f(x, y) = \begin{bmatrix} x + y^2 \\ xy \end{bmatrix}$.

Suppose $g : \mathbf{R}^2 \to \mathbf{R}^2$ is a map such that $g(0, 0) = (3, 1)$ and such that

$$Dg(0, 0) = \begin{bmatrix} 7 & 1 \\ 1 & 0 \end{bmatrix}.$$

Let $h = f \circ g$. Find $Dh(0, 0)$.