Math 51 - Autumn 2010 - Midterm Exam I

Name: ______________________________

Student ID: ______________________________

Select your section:

Brandon Levin 05 (1:15-2:05) 15 (11:00- 11:50)	Amy Pang 14 (10:00-10:50) 17 (1:15-2:05)	Yuncheng Lin 06 (1:15-2:05) 21 (11:00-11:50 AM)	Rebecca Bellovin 09 (11:00-11:50) 23 (1:15-2:05)
Xin Zhou 02 (11:00-11:50) 08 (10:00- 10:50)	Simon Rubinstein-Salzedo 18 (2:15-3:05) 24 (1:15-2:05)	Frederick Fong 20 (10:00-10:50) 03 (11:00-11:50)	Jeff Danciger ACE (1:15-3:05)

Signature: ______________________________

Instructions:

- Print your name and student ID number, select your section number and TA's name, and **sign above to indicate that you accept the Honor Code**.
- There are nine problems on the pages numbered from 1 to 9, and each problem is worth 10 points. Please check that the version of the exam you have is complete and correctly stapled.
- Read each question carefully. In order to receive full credit, please show all of your work and justify your answers unless specifically directed otherwise.
- You do not need to simplify your answers unless specifically instructed to do so. If you use a result proved in class or in the text, you must clearly state the result before applying it to your problem.
- You have 2 hours. This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted. If you finish early, you must hand your exam paper to a member of the teaching staff.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
- It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner.

Problem 1.

Show that if $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is a linearly independent set, then $\{\mathbf{u} - \mathbf{v}, \mathbf{u} + 2\mathbf{v}, \mathbf{w}\}$ is a linearly independent set.

Problem 2.
Find all solutions to the following system of equations:

$$x + y = 1, \qquad 2x + 2z = 4, \qquad 2x + y + z = 3.$$

Problem 3. Identify each of the statements as always true (**T**) or sometimes false (**F**). You do not need to justify your answers.

a) If A is a 13×17 matrix with $\dim(N(A)) = 4$, then the column space of A is $\mathbf{R^{13}}$.

b) If A is a 12×16 matrix with $\dim(C(A)) = 11$, then the nullspace has a basis of 5 vectors in $\mathbf{R^{12}}$.

c) For any matrix A, $\dim(C(A)) = \dim(C(rref(A)))$.

d) If A is a 14×12 matrix with $\dim(C(A)) = 10$, then the number of vectors in $N(A)$ is 2.

e) If $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is a linearly dependent set of vectors in $\mathbf{R^3}$, then $\dim(\text{span}(\mathbf{v}, \mathbf{w})) = 2$.

Problem 4. Let c be a scalar. Find a basis for the null space of the matrix A:

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 & c \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

.

Problem 5. Suppose that $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors in $\mathbf{R^n}$. Show that $||\mathbf{u}|| = ||\mathbf{v}||$ if and only if $\mathbf{u} + \mathbf{v}$ is orthogonal to $\mathbf{u} - \mathbf{v}$.

Problem 6.
Suppose that $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbf{R^3}$ which satisfy the following:

$$\mathbf{v} \cdot \mathbf{w} = 0 \qquad ||\mathbf{v} \times \mathbf{u}|| > 0 \qquad ||\mathbf{w}|| > ||\mathbf{v}||.$$

Complete each of the expressions below with "$>$", "$<$", or "$=$", if you can. If there is not enough information to decide, write, "?". You do not need to prove your answers.

(a) $||\mathbf{v}||$ $\qquad$ 0

(b) $||\mathbf{v} \times \mathbf{w} - \mathbf{w} \times \mathbf{v}||$ $\qquad$ 0

(c) $||\mathbf{u}||||\mathbf{v}||$ $\qquad$ $||\mathbf{u} \times \mathbf{v}||$

(d) $||\mathbf{v} \times (\mathbf{u} \times \mathbf{w})||$ $\qquad$ $||(\mathbf{v} \times \mathbf{u}) \times \mathbf{w}||$

(e) $||\mathbf{v} \times \mathbf{w} + \mathbf{w} \times \mathbf{v}||$ $\qquad$ 0

Problem 7.
Let $\{\mathbf{u}, \mathbf{v}\}$ be a linearly independent set of vectors in $\mathbf{R^3}$, and let C be the set of vectors in $\mathbf{R^3}$ which are orthogonal to $\mathbf{u} \times \mathbf{v}$:

$$C = \{\mathbf{w} \mid \mathbf{w} \cdot (\mathbf{u} \times \mathbf{v}) = 0\}.$$

a) Show that C is a linear subspace.

b) Find a basis for C.

Problem 8. Let $\mathbf{T} : \mathbf{R^2} \to \mathbf{R^2}$ be the linear transformation which satisfies

$$\mathbf{T}\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } \mathbf{T}\left(\begin{bmatrix} 0 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}.$$

Find the matrix A such that $\mathbf{T}$ is equivalent to multiplication by A.

Problem 9. Let $\{\mathbf{u}, \mathbf{v}\}$ be a linearly independent set of vectors in $\mathbf{R^n}$, and let A be the matrix

$$A = \begin{bmatrix} \mathbf{u} \cdot \mathbf{u} & \mathbf{u} \cdot \mathbf{v} \\ \mathbf{u} \cdot \mathbf{v} & \mathbf{v} \cdot \mathbf{v} \end{bmatrix}$$

What is the rank of A? What is the nullity of A?

The following boxes are strictly for grading purposes. Please do not mark.

Question	Score	Maximum
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total		90