Math 51- Winter 2008 - Midterm Exam II

Please circle the name of your TA:

Zachary Cohn José Perea Nikola Penev Man Chun Li

Daniel Mathews Theodora Bourni Anssi Lahtinen Isidora Milin

Circle the time your TTh **section** meets: 10:00 11:00 1:15 2:15

Your name (print):

Student ID:

Sign to indicate that you accept the honor code:

Instructions: Circle your TA's name and the time that you attend the TTh section. Read each question carefully, and **show all your work**. You have 90 minutes to do all the problems.

During the test, **you may NOT use any notes, books, or calculators**.

Question	1	2	3	4	5	6	Total
Maximum	20	16	12	22	16	14	100
Score							

Problem 1. (20 points total) Consider the matrix $A = \begin{pmatrix} 3 & 2 \\ -2 & -2 \end{pmatrix}$

(a)(10 points) find the eigenvalues and the corresponding eigenvectors of A.

(b) (3pts) what are the eigenvalues of A^{99}? (A is the matrix given above)

(c) (3pts) is A^{99} diagonalizable?

(d) (4pts) if R is a region in $\mathbb{R}^2$ of area 4, what is the area of its image under the linear transformation with associated matrix A?

Problem 2. (16 pts total) Consider the linear transformation T that reflects vectors in $\mathbb{R}^2$ across the line $y = 2x$.

(a)(8 pts) find the matrix A corresponding to this linear transformation.

(b)(8 pts) what are the eigenvalues and eigenvectors of A?

Problem 3. (12 points) Evaluate the following limit, or explain why the limit fails to exist.

$$\lim_{x,y\to 0} \frac{x^2y^2}{x^4+y^4} =$$

Problem 4. (22 points total) Assume you are standing at a point P of coordinates $x = 20$, $y = 10$ on a hillside whose height (in feet above sea level) is given by

$$h(x, y) = 500 - x^2 + 2xy + 3y^2,$$

where x points E (east), and y points N (north).

(a) (6pts) suppose you start moving in the SW direction, do you ascent or descend?

(b) (8pts) find the equation of the tangent plane to the graph of $h(x, y)$ at the point P.

(c) (8pts) use the result in (b) to approximate the change in height you experience if you move from P to the point of coordinates $x = 21$, $y = 9$.

Problem 5. (16 pts total) Consider A and B two 4×4 matrices with $\det A = 3$ and $\det B = 2$. Furthermore, denote by $T : \mathbb{R}^4 \to \mathbb{R}^4$ the linear transformation $T(x) = Ax$ for all $x \in \mathbb{R}^4$.

(a) (3 pts) find $\det(AB)$

(b) (3 pts) find $\det A^{-1}$

(c) (3 pts) find $\det(2A)$

(e) (3pts) is the linear transformation T defined above invertible?

(d) (4pts) can we find a basis $\mathcal{B}$ of $\mathbb{R}^4$ in which T has matrix B?

Problem 6. (14 points) Assume $\mathbf{x}(t)$ is the position vector at time t of an ant moving smoothly on a sphere of radius 5 centered at the origin. Prove that, at any moment, the velocity vector $\dfrac{d\mathbf{x}}{dt}$ of the ant is perpendicular to its position vector.