Math 51- Winter 2008 - Midterm Exam I

Please circle the name of your TA:

Zachary Cohn	José Perea	Nikola Penev	Man Chun Li
Daniel Mathews	Theodora Bourni	Anssi Lahtinen	Isidora Milin

Circle the time your TTh **section** meets: 10:00 11:00 1:15 2:15

Your name (print):

Student ID:

Sign to indicate that you accept the honor code:

Instructions: Circle your TA's name and the time that you attend the TTh section. Read each question carefully, and show all your work. You have 90 minutes to do all the problems.

During the test, **you may NOT use any notes, books, or calculators**.

Question	1	2	3	4	5	6	7	8	Total
Maximum	12	12	12	15	12	15	12	10	100
Score									

Formulas you may use:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \times \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} x_2y_3 - x_3y_2 \\ x_3y_1 - x_1y_3 \\ x_1y_2 - x_2y_1 \end{pmatrix}$$

Problem 1. (12 points total)

(a) Write the equation of the line passing through the point $(2, 1)$ and with normal vector $\mathbf{n} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

(b) What is the parametric equation of this line?

Problem 2. (12 points) Given two vectors u and v such that $\|u\| = \|v\|$ show that $u + v$ and $u - v$ are orthogonal (perpendicular). (Hint: Use dot product)

Problem 3. (12 points) Consider system of equations

$$\begin{cases} x + y = 2 \\ x + ay = b. \end{cases}$$

where a and b are constants, and x and y are the unknowns. Determine all the values of a and b for which the system above has:

(a) no solution ____________________________________

(b) a unique solution ____________________________________

(c) exactly two solution ____________________________________

(d) more than two solutions ____________________________________

(**Explain your answer below!**)

Problem 4. (15 points total) Let $\{u, v, w\}$ be three linearly independent vectors.

(a) (5 points) Show that $\{u + v, u - v\}$ are linearly independent.

(b) (5 points) Are $\{u, u + v, u - v\}$ linearly independent? Please explain your answer.

(c) (5 points) Is w in the span of $\{u + w, v + w\}$? Please explain your answer.

Problem 5. (15 points total) Consider the matrix $A = \begin{pmatrix} 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$

(a) find a basis for $N(A)$

(b) find a basis for $C(A)$

(c) given that $A \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$, find all solutions to the equation $Ax = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

Problem 6. (12 points total) Which of the following are linear subspaces of $\mathbb{R}^2$? Please explain your answer.

(a) the set $V = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1 \leq 0\}$

(b) the set $V = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1 = 0\}$

Problem 7. (12 pts total) Consider the following linear subspace of $\mathbb{R}^3$:

$$V = \{(x_1, x_2, x_3) \mid x_1 - x_2 + x_3 = 0\}.$$

(a) find a basis for V

(b) give an example of a matrix A such that $N(A) = V$.

(c) give an example of a matrix A such that $C(A) = V$.

Problem 8. (10 points) Circle T or F to mark each of the following true or false. Expanations are not required for this problem.

(a) the dot product of two vectors in $\mathbb{R}^3$ is a vector in $\mathbb{R}^3$ **T F**

(f) any three vectors in $\mathbb{R}^3$ span $\mathbb{R}^3$. **T F**

(b) a system of 3 linear equations with 5 unknowns cannot have a unique solution. **T F**

(c) a system of 5 linear equations with 3 unknowns cannot have more then one solution. **T F**

(d) for all matrices A, the column space of A equals the column space of the rref (A) **T F**

(e) for all matrices A, the null space of A equals the null space of the rref (A). **T F**

(g) if A is a 4×2 matrix then $\dim N(A) \leq 2$ **T F**

(h) if A is a 2×4 matrix then $\dim N(A) \geq 2$ **T F**

(i) there are 3×6 matrices with $\dim N(A) = 3$ and $\dim C(A) = 3$. **T F**

(j) there are 6×3 matrices with $\dim N(A) = 3$ and $\dim C(A) = 3$. **T F**