MATH 51 MIDTERM 2

November 13, 2008

Professor: Han Kargin White Wise **TTh Section Number:** [ ]

	Olena Bormashenko	Luis Diogo	Kaveh Fouladgar	Frederick Tsz Ho Fong
TA:	Robin Koytcheff	Jason Lo	Jonathan Lee	Jose Perea
		Josh Genauer		

Time your TTh **section** meets: morning afternoon

Your name (print): Student ID:

Sign to indicate that you accept the honor code:

Instructions: Circle your professor's name, your TA's name, and the time that you attend the TTh section. During the test, you may not use notes, books, or calculators. Read each question carefully, and show all your work. Each of the nine problems is worth 10 points. You have 90 minutes to do all the problems.

1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Name:

1. Find the following.

1(a). The matrix for for the linear map T given by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 7x - 2y \\ x + 3y \\ 5y \end{bmatrix}$.

1(b). The matrix for reflection in $\mathbf{R}^2$ about the line $y = -x$.

1(c). The matrix for $T : \mathbf{R}^3 \to \mathbf{R}^3$, where T is rotation by 180° about the y-axis, followed by rotation by 180° about the z-axis.

Name:

2. Find the determinant of the matrix $C = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & 1 \\ 2 & 1 & 1 \end{bmatrix}$.

Name:

3. Find the inverse of the matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

Name:

4(a). Find all eigenvalues of the matrix $A = \begin{bmatrix} 1 & 4 & 3 \\ 1 & 1 & 2 \\ 0 & 0 & 7 \end{bmatrix}$.

Name:

4(b). The matrix $B = \begin{bmatrix} -1 & 8 \\ 8 & 11 \end{bmatrix}$ has characteristic polynomial $p(\lambda) = (\lambda - 15)(\lambda + 5)$. Find a basis of $\mathbf{R}^2$ consisting of eigenvectors of B.

Name:

4(c). Suppose that C is a **symmetric** 2x2 matrix with determinant 7. Suppose that the vector $\mathbf{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ is an eigenvector of C with eigenvalue 5. Find an eigenvector $\mathbf{w}$ that is **not** a scalar multiple of $\mathbf{v}$, and find its eigenvalue. Explain.

Name:

5. Consider the basis $\mathcal{B}$ of $\mathbf{R}^2$ consisting of $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$.

5(a). Find the matrix C such that $\mathbf{w} = C[\mathbf{w}]_{\mathcal{B}}$ for all vectors $\mathbf{w} \in \mathbf{R}^2$.

5(b). Find a matrix M so that $[\mathbf{w}]_{\mathcal{B}} = M\mathbf{w}$ for all $\mathbf{w} \in \mathbf{R}^2$.

Name:

5(c). Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ the the linear map such that $T(\mathbf{v}_1) = 5\mathbf{v}_1 + \mathbf{v}_2$ and $T(\mathbf{v}_2) = 7\mathbf{v}_1$. Find the matrix B for T in the coordinate system determined by $\mathcal{B}$. In other words, find a matrix B such that

$$[T(\mathbf{w})]_{\mathcal{B}} = B[\mathbf{w}]_{\mathcal{B}}$$

for all $\mathbf{w} \in \mathbf{R}^2$.

5(d). Let A be the matrix that represents T in standard Cartesian coordinates. Write an equation that expresses A in terms of B, C, and M:

$$A =$$

(You do not need to calculate A.)

Name:

6. Let V be the subspace of $\mathbf{R}^3$ spanned by $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

Consider the coordinate system for V determined by the basis $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$.

6(a). Find the vector $\mathbf{w} \in \mathbf{R}^3$ whose expression in the $\mathcal{B}$-coordinate system is

$$[\mathbf{w}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

Name:

6(b). Find $[\mathbf{v}]_{\mathcal{B}}$ (the expression for $\mathbf{v}$ in the $\mathcal{B}$-coordinate system) for the vector

$$\mathbf{v} = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}.$$

Name:

7(a). Find the equation of the tangent plane to the surface $z = x^2y + y^3$ at the point $(x, y, z) = (2, 1, 5)$.

7(b). Find the matrix $DG(x, y)$ for the map G given by $G(x, y) = \begin{bmatrix} x^2y \\ y^3 \\ x \sin y \end{bmatrix}$.

Name:

8. Suppose that $F : \mathbf{R}^2 \to \mathbf{R}^2$ is a map such that $F(0,0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and such that $DF(0,0) = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$.

8(a). Estimate $F(.002, .003)$.

8(b). Find a point (x, y) near $(0,0)$ so that $F(x,y) \simeq \begin{bmatrix} 3.004 \\ 1.007 \end{bmatrix}$.

Name:

9(a). Suppose $\mathbf{v}$ is an eigenvector of A with eigenvalue λ. Prove that $\mathbf{v}$ is also an eigenvector of $I + A^2$, and find its eigenvalue.

9(b). Suppose that A and B are similar matrices, i.e., that $B = C^{-1}AC$ for some invertible matrix C. Suppose that λ is a real number. Prove that $\lambda I - A$ and $\lambda I - B$ are also similar.