MATH 51 MIDTERM 1

October 16, 2008

Professor: Han Kargin White Wise **TTh Section Number:** [ ]

TA:				
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Time your TTh **section** meets: morning afternoon

Your name (print): Student ID:

Sign to indicate that you accept the honor code:

Instructions: Circle your professor's name, your TA's name, and the time that you attend the TTh section. During the test, you may not use notes, books, or calculators. Read each question carefully, and show all your work. Each of the nine problems is worth 10 points. You have 90 minutes to do all the problems.

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Total	

Name:

1. Find all solutions of the following system:

$$\begin{array}{ccccccccc} x_1 & + & 2x_2 & + & x_3 & + & x_4 & = & 7 \\ x_1 & + & 2x_2 & + & 2x_3 & - & x_4 & = & 12 \\ 2x_1 & + & 4x_2 & & & & 6x_4 & = & 4. \end{array}$$

Name:

2(a). Find a parametric equation for the plane containing the points $A = (1, 2, 3)$, $B = (4, 5, 6)$, and $C = (2, 2, 3)$.

2(b). Find the equation for the plane that passes through the point $A = (1, 2, 3)$ and that is perpendicular to the vector $\mathbf{v} = \begin{bmatrix} 7 \\ 3 \\ 5 \end{bmatrix}$. (Your answer should be an equation of the form $ax + by + cz = d$.)

Name:

3(a) Suppose that Δ is an equilateral triangle in $\mathbf{R}^3$ and that the edges of Δ each have length 1. Let A, B, and C be the vertices of Δ. Find

$$(3\overrightarrow{AB}) \cdot (5\overrightarrow{AC}).$$

Name:

3(b). Suppose $\mathbf{A} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$ are orthogonal vectors in $\mathbf{R}^4$ with $a_4 > 0$ and $b_4 > 0$.

Let $\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$.

Prove that the angle between **a** and **b** is obtuse (i.e., greater than $\pi/2$).

Name:

4. Let V be the set of vectors in $\mathbf{R}^4$ that are orthogonal to the vector $\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 5 \end{bmatrix}$.

Find a basis for V.

Name:

5. Are the following three vectors in $\mathbf{R}^3$ linearly independent or linearly dependent? Show your work and explain your answer.

$$\mathbf{u} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} 4 \\ 2 \\ 9 \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} -2 \\ -2 \\ 3 \end{bmatrix}.$$

Name:

6. Let

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 3 & 3 \\ 1 & 1 & -1 \end{bmatrix}.$$

6(a). What condition(s) must $\mathbf{b}$ satisfy to be in the column space of A?

(Your answer should be one or more equations of the form $?\,b_1 + ?\,b_2 + ?\,b_3 + ?\,b_4 = ?$.)

Name:

6(b) Find a matrix M such that the column space of A is equal to the null space of M. [Hint: use your answer to part (a).]

Name:

7(a) Suppose $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ are linearly independent vectors in $\mathbf{R}^n$. Prove that the vectors $\mathbf{x}+\mathbf{y}$, $\mathbf{x}-\mathbf{y}$, and $\mathbf{x}+\mathbf{y}+\mathbf{z}$ are also linearly independent.

Name:

7(b) Suppose that $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are linearly dependent vectors in $\mathbf{R}^n$, and that $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly independent.

Prove that $\mathbf{v}_4$ is a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$.

Name:

8. Let A be the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 3 \\ 1 & 1 & 2 & 1 & 3 \\ 0 & 0 & 0 & 2 & -2 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix}$$

The reduced echelon form for A is

$$R = \begin{bmatrix} 1 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(You do not need to check this.)

8(a) (3 points) Find a basis for the column space $C(A)$ of A.

Name:

8(b) (4 points) Find a basis for the nullspace $N(A)$ of A.

Name:

8(c) (3 points) Find all solutions $\mathbf{x}$ of

$$A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}.$$

[Hint: compare the right hand side of this equation to the columns of A.]

Name:

9(a,b,c). Suppose V is a set of vectors in $\mathbf{R}^n$. What three properties must V have in order to be a linear subspace of V?

9(d,e). Suppose that V is a linear subspace of $\mathbf{R}^n$ and that $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are vectors in $\mathbf{R}^n$. What two properties must $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ have in order to be a basis for V?