

MIDTERM 1

- Complete the following problems. You may use any result from class you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted.
- In order to receive full credit, please show all of your work and justify your answers. You do not need to simplify your answers unless specifically instructed to do so.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
- Please sign the following:
“On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination.”

Name: _____

Signature: _____

The following boxes are strictly for grading purposes. Please do not mark.

1	15 pts	
2	10 pts	
3	15 pts	
4	15 pts	
5	15 pts	
6	10 pts	
7	15 pts	
Total	95 pts	

(1) (15 points) Complete each of the following sentences.

(a) A collection of vectors $\vec{v}_1, \dots, \vec{v}_m$ is defined to be linearly dependent if

(b) The dimension of a subspace W is defined to be

(c) The book lists 4 properties a matrix must have to be in reduced row echelon form. Two of these properties are

(i)

(ii)

(2) (10 points) Compute the reduced row echelon form of the matrix

$$\begin{pmatrix} 1 & 3 & 4 & 0 \\ 1 & -1 & 0 & 1 \\ 2 & 3 & 5 & 3 \\ -2 & -1 & -3 & 1 \end{pmatrix}.$$

(3) (15 points) A matrix B and its reduced row echelon form are given below:

$$B = \begin{pmatrix} 1 & 2 & 4 & 11 & 7 \\ 2 & 4 & 5 & 13 & 8 \\ 3 & 6 & 6 & 15 & 10 \end{pmatrix} \quad \text{and} \quad \text{rref}(B) = \begin{pmatrix} 1 & 2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

(a) Give a basis for the null space of B . You do not need to prove that your collection is a basis.

(b) Give a basis for the column space of B . You do not need to prove that your collection is a basis.

(c) Find a non-zero vector orthogonal to all three of the vectors $\begin{pmatrix} 1 \\ 2 \\ 4 \\ 11 \\ 7 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 4 \\ 5 \\ 13 \\ 8 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 6 \\ 6 \\ 15 \\ 10 \end{pmatrix}$.

- (4) (15 points) Determine whether each statement is true or false and circle your answer. In all these statements, A is an arbitrary $m \times n$ matrix and $R = \mathbf{rref}(A)$. No justification is necessary for your answers.

T or F (a) $N(A) = N(R)$.

T or F (b) $C(A) = C(R)$.

T or F (c) The dimensions of $C(A)$ and $C(R)$ are the same.

T or F (d) If the equation $A\vec{x} = \vec{0}$ has infinitely many solutions, then for any $\vec{b} \in \mathbb{R}^m$ the system $A\vec{x} = \vec{b}$ has infinitely many solutions.

T or F (e) If $n > m$, it is possible for $\dim(C(A)) = m$.

T or F (f) If $n > m$, it is possible for $\dim(C(A)) = n$.

T or F (g) If $n > m$, it is possible for $\dim(N(A)) = 0$.

- (5) (15 points) Let P_1 and P_2 be two planes passing through the point $(1, 1, 1)$ such that \vec{n}_1 is normal to P_1 and \vec{n}_2 is normal to P_2 , where

$$\vec{n}_1 = \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix} \quad \text{and} \quad \vec{n}_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}.$$

- (a) Find the parametric equation of the line ℓ which is the intersection of P_1 and P_2 .

- (b) Prove that P_1 is *not* a subspace. (Hint: To answer this question you should find a property of subspaces that P_1 does not have. Be sure you give full justification for any claims you make.)

- (6) (10 points) Suppose that \vec{x} and \vec{y} are two vectors in \mathbb{R}^n with equal magnitude. Prove that the vectors $\vec{x} + \vec{y}$ and $\vec{x} - \vec{y}$ are orthogonal.

(7) (15 points) Consider a linear transformation $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ which satisfies

$$\begin{aligned} T\left(\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}\right) &= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, & T\left(\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}\right) &= \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \\ T\left(\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}\right) &= \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, & \text{and } T\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\right) &= \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}. \end{aligned}$$

(a) Compute $T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right)$.

(b) Suppose you are told that a linear transformation S satisfies

$$S\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, S\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \text{ and } S\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 10 \\ 15 \\ -5 \end{bmatrix}.$$

What is the matrix A that satisfies $A\vec{x} = S(\vec{x})$ for all $\vec{x} \in \mathbb{R}^3$?