Math 51 - Autumn 2007 - Midterm Exam II

Name: ____________________

Student ID: ____________________

Select your section:

Kirsten Wickelgren 03 (11:00-11:50 AM) 20 (10:00-10:50AM)	Jon Lee 08 (11:00-11:50AM) 26 (10:00-10:50AM)	Jesse Gell-Redman 11 (1:15-2:05PM) 24 (2:15-3:05PM)	Jason Miller 23 (1:15-2:05PM) 30 (2:15-3:05PM)	Josh Genauer 12 (1:15-2:05PM) 15 (11:00-11:50AM)
Dmitriy Ivanov 14 (11:00-11:50AM) 29 (1:15-2:05PM)	Olena Bormashenko 17 (1:15-2:05PM) 21 (11:00-11:50AM)	Penka Georgieva 18 (1:15-2:05PM) 27 (11:00-11:50AM)	Robin Koycheff 02 (11:00-11:50 AM) 05 (1:15-2:05PM)	Jack Hall 09 (11:00-11:50AM) 06 (1:15-2:05PM)
Andres Angel MATH 51				

Signature: ____________________

Instructions: Print your name and student ID number, print your section number and TA's name, write your signature to indicate that you accept the honor code. During the test, you may not use notes, books, calculators. Read each question carefully, and show all your work.
There are ten problems on the pages numbered from 1 to 10, with the total of 100 points. Point values are given in parentheses. You have 2 hours (until 9PM) to answer all the questions.

In the exam all vectors are columns, but sometimes we use transpose to write them horizontally.

Thus $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_k \end{bmatrix} = [v_1, v_2, \ldots, v_k]^T$.

Similarly $\vec{v}^T$ is a row $[v_1, v_2, \ldots, v_k]$.

The dot product of two vectors is denoted as $\vec{v} \cdot \vec{w}$.

Problem 1. (10 pts.) a) Write the definition of when λ is an eigenvalue of A.

b) Let A be the following 3×3 matrix:

$$A = \begin{bmatrix} \star & 2 & \star \\ 1 & -3 & 0 \\ \star & -1 & \star \end{bmatrix}$$

Where $\star$ denotes the entries of A that are unknown.

Assuming that the vector $\vec{v} = \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix}$ is an eigenvector of A, find the corresponding eigenvalue.

Problem 2. (10 pts.) Let A be the following matrix:

$$A = \begin{bmatrix} 0 & -2 & -1 \\ 2 & 4 & 1 \\ -2 & -2 & 1 \end{bmatrix}$$

Find a basis of the eigenspace corresponding to the eigenvalue $\lambda = 2$.

Problem 3. (10 pts.) Find the inverse of the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 0 & -1 \\ 0 & 2 & 1 \end{bmatrix}$$

b) For the same matrix A find all solutions to the equation

$$A^{-1}\vec{x} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Problem 4. (10 pts.) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear map:

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + 2y \\ -x + y + z \\ -2x - y + z \end{bmatrix}$$

a) Find the derivative DT of the map T.

b) Find the inverse of T or show that this inverse does not exist.

Problem 5. (10 pts.) A 3×3 matrix M has two linearly independent eigenvectors corresponding to eigenvalue $\lambda = 4$ and one eigenvector corresponding to eigenvalue $\lambda = -1$.

a) Is M diagonalizable? Explain your answer.

b) Find $\det M$.

Problem 6. Find an equation of the tangent plane to the graph of the function

$$f(x,y) = x^3y^2 - y^2x + x^2 + 1$$

at the point $(1,1,2)$.

Problem 7. (10 pts.) Let $Q(x, y) = x^2 - 2axy + y^2$

a) For what values of the parameter a is the quadratic form Q positive definite?

b) For what values of the constant a does the function $Q(x, y)$ satisfy the following expression:

$$\frac{\partial^2 Q}{\partial x \partial y} = \frac{\partial^2 Q}{\partial x^2}$$

Problem 8. (10 pts.) Let $\vec{x} : \mathbb{R} \to \mathbb{R}^2$ be defined by:

$$\vec{x}(t) = \begin{bmatrix} 2\cos t \\ \sin t \end{bmatrix}$$

and $f : \mathbb{R}^2 \to \mathbb{R}$ be given by $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = x^2 + y^2$. Find the derivative of the composition $f\left(\vec{x}(t)\right)$.

Problem 9. (10 pts.) Let P be the xz- plane in $\mathbb{R}^3$.

a) Let T_1 be the linear transformation $T_1 : \mathbb{R}^3 \to \mathbb{R}^3$ which reflects every vector across the plane P. What are the eigenvectors and eigenvalues of T_1?

b) Let T_2 be the linear transformation $T_2 : \mathbb{R}^3 \to \mathbb{R}^3$ which orthogonally projects every vector to the plane P. What are the eigenvectors and eigenvalues of T_2?

Problem 10. (10 pts.) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function defined by

$$f(x,y) = \begin{cases} \dfrac{(x^3y + y^3x)}{(x^4 + y^4)} & \text{for } (x,y) \neq (0,0) \\ a & \text{for } (x,y) = (0,0) \end{cases}$$

What, if any, value of a will make $f(x,y)$ a continuous function?

Question	Score	Maximum
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total		100