MATH 51 MIDTERM II November 16, 2006

Instructions:

- No calculators, books, notes, or electronic devices may be used during the exam.
- You have 90 minutes.
- There are 9 problems, many with multiple parts. Many questions have short answers requiring no computation. The point value of each part of each problem is indicated in brackets at the beginning of that part. You should work quickly so as to not leave out problems towards the end of the exam.
- Show computations on the exam sheet. If extra space is needed use the back of a page.

Name:________________________________
(print clearly)

Signature:________________________________
(for acceptance of honor code)

Problem 1 (10 points)	
Problem 2 (10 points)	
Problem 3 (12 points)	
Problem 4 (10 points)	
Problem 5 (12 points)	
Problem 6 (13 points)	
Problem 7 (13 points)	
Problem 8 (15 points)	
Problem 9 (5 points)	
Total (100 points)	

Your TA/discussion section (circle one):

Antebi (15, 18)	Ayala (3, 6)	Easton (14, 17)
Fernanadez (2, 5)	Kim (8, 11)	Koytcheff (9, 12)
Lo (21, 24)	Rosales (26, 27)	Tzeng (20, 23)
Zamfir (29, 30)	Schultz (51A)	

1. (a) [2] If A is an $m \times n$ matrix (m rows, n columns) so that the transformation $T(\mathbf{x}) = A\mathbf{x}$ is one-to-one, what is rank(A)? (Circle one.)

(i) m (ii) n (iii) not enough information

(b) [2] If B is an $m \times n$ matrix so that the transformation $T(\mathbf{x}) = B\mathbf{x}$ is onto, what is rank(B)? (Circle one.)

(i) m (ii) n (iii) not enough information

(c) [6] Find the inverse of $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -3 \\ 1 & 2 & 2 \end{bmatrix}$.

2. (a) [6] Let $V = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -7 \\ 5 \\ 4 \end{bmatrix} \right\}$. Find an orthonormal basis for V. [Note the first two vectors given are orthogonal.]

(b) [4] If $\{\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3\}$ is an orthonormal set of vectors and if $\mathbf{w} \in \text{span}\{\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3\}$, explain why $\mathbf{w} = (\mathbf{w} \cdot \mathbf{n}_1)\mathbf{n}_1 + (\mathbf{w} \cdot \mathbf{n}_2)\mathbf{n}_2 + (\mathbf{w} \cdot \mathbf{n}_3)\mathbf{n}_3$. [Hint: start with $\mathbf{w} = c_1\mathbf{n}_1 + c_2\mathbf{n}_2 + c_3\mathbf{n}_3$.]

3. Let $\mathbf{v}_1 = \begin{bmatrix} 7 \\ 5 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 4 \\ 3 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 6 \\ 5 \end{bmatrix}$, $\mathbf{v}_4 = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 4 \end{bmatrix}$. Let $T : \mathbf{R}^4 \to \mathbf{R}^4$ be a linear transformation satisfying:

$$\begin{aligned} T(\mathbf{v}_1) &= 4\mathbf{v}_1 + 2\mathbf{v}_2 - 3\mathbf{v}_3 + \mathbf{v}_4 \\ T(\mathbf{v}_2) &= 2\mathbf{v}_2 + \mathbf{v}_3 - 5\mathbf{v}_4 \\ T(\mathbf{v}_3) &= 3\mathbf{v}_3 - 7\mathbf{v}_4 \\ T(\mathbf{v}_4) &= \mathbf{v}_4 \end{aligned}$$

(a) [4] Find the matrix, B, for the transformation T relative to the basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ of $\mathbf{R}^4$.

(b) [4] Give the change of basis matrix, C, so that the matrix, A, of the transformation T relative to the standard basis of $\mathbf{R}^4$ is given by $A = CBC^{-1}$.

(c) [4] Find $\det(B)$ and $\det(A)$.

4. Suppose B and C are $n \times n$ matrices with C invertible.

   (a) [5] If $\mathbf{v}$ is an eigenvector of B show that $C\mathbf{v}$ is an eigenvector of $A = CBC^{-1}$.

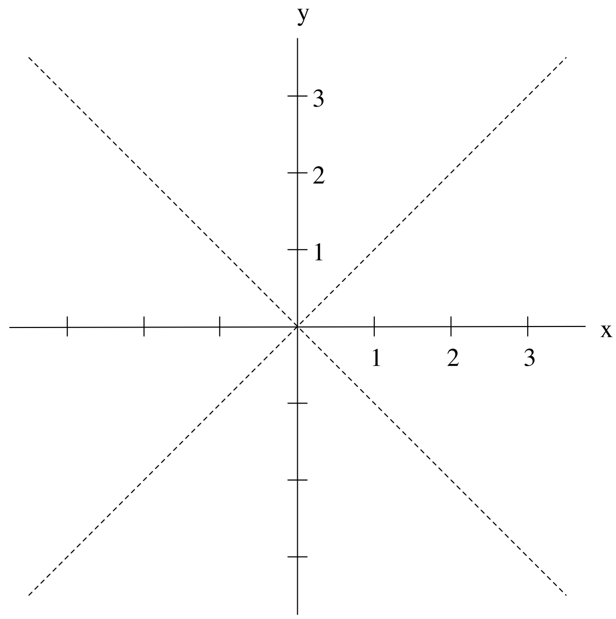
   (b) [5] Show that B and $A = CBC^{-1}$ have the same eigenvalues.

5. (a) [5] Find the eigenvalues of $\begin{bmatrix} 3 & 4 \\ 1 & 3 \end{bmatrix}$.

(b) [7] Find a basis for $\mathbf{R}^2$ consisting of eigenvectors of the matrix $\begin{bmatrix} 3 & 4 \\ 1 & 3 \end{bmatrix}$ from part (a).

6. Let $f : \mathbf{R}^2 \to \mathbf{R}$ be the function defined by $f(x,y) = \begin{cases} |x| & \text{if } |x| \geq |y| \\ |y| & \text{if } |x| \leq |y| \end{cases}$.

(a) [6] Sketch the level curves of $f(x,y)$ for $c = 1,\ 2,\ 3$. Indicate in your picture which level curve is which. (The dashed lines are the points where $|x| = |y|$.)



(b) [3] At which points is $f(x,y)$ continuous? (No explanation needed.)

(c) [4] Circle the value of the partial derivative $\frac{\partial f}{\partial x}$ at each of the given points.

i. $\frac{\partial f}{\partial x}(0,0) =$ 1 0 -1 does not exist

ii. $\frac{\partial f}{\partial x}(2,0) =$ 1 0 -1 does not exist

iii. $\frac{\partial f}{\partial x}(2,2) =$ 1 0 -1 does not exist

iv. $\frac{\partial f}{\partial x}(0,2) =$ 1 0 -1 does not exist

7. Compute the following limits or give a reason the limit does not exist.

(a) [4] $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{1+2x^2+3y^2+4z^2}{1-x^2-y^2-z^2}$

(b) [4] $\displaystyle\lim_{(x,y)\to(2,2)} \frac{x^2-y^2}{x-y}$

(c) [5] $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x+y}{\sqrt{x^2+y^2}}$

8. Let $f(x,y,z) = 2x^3y^2z + xy^2z^2 + e^{2y-2}z$.

(a) [6] Compute $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and $\frac{\partial f}{\partial z}$.

(b) [3] Calculate $\nabla f = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z})$ at the point $(1,1,1)$.

(c) [2] Calculate $f(1,1,1)$. [Recall $e^0 = 1$.]

(d) [4] Find the best approximation to $f(x,y,z)$ near $(1,1,1)$ of the form

$$L(x,y,z) = d + a(x-1) + b(y-1) + c(z-1).$$

9. [5] Let $\mathbf{g} : \mathbf{R}^2 \to \mathbf{R}^2$ and $\mathbf{h} : \mathbf{R}^2 \to \mathbf{R}^2$ be differentiable functions satisfying:

$$\mathbf{h}(1,2) = (3,5), \quad \mathbf{h}(3,5) = (1,2), \quad \mathbf{g}(1,2) = (3,5), \quad \mathbf{g}(3,5) = (1,2),$$

$$D\mathbf{h}(1,2) = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}, \quad D\mathbf{h}(3,5) = \begin{bmatrix} 1 & 1 \\ 3 & 5 \end{bmatrix},$$

$$D\mathbf{g}(1,2) = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}, \quad D\mathbf{g}(3,5) = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}.$$

Calculate $D(\mathbf{h} \circ \mathbf{g})$ at the point $(1,2)$. [You may not need all the data given.]