MATH 51 MIDTERM II

November 17, 2005

Name: ______________________________

Numeric Student ID: ____________________

Instructor's Name: _____________________

I agree to abide by the terms of the honor code:

Signature: ___________________________

Instructions: Print your name, student ID number and instructor's name in the space provided. During the test you may not use notes, books or calculators. Read each question carefully and **show all your work**; full credit cannot be obtained without sufficient justification for your answer unless explicitly stated otherwise. Underline your final answer to each question. There are 9 questions. You have 90 minutes to do all the problems.

Question	Score	Maximum
1		10
2		6
3		8
4		6
5		10
6		10
7		15
8		10
9		10
Total		85

1. Consider the function

$$f(x, y) = x^4 y^3$$

and the point $P = (1, 1, 1)$ on its graph.

(a) Write down the equation of the tangent plane at the graph of the function at the point P.

(b) Using your answer from (a), write down an expression for the change, Δz, in $z = f(x, y)$ depending on Δx and Δy, the change in x and y, respectively, near the point $P = (1, 1, 1)$. Is the function $f(x, y)$ more sensitive to a change in x or to a change in y? Explain.

(c) Using your answer to (b), find the approximate value of $f(1.01, 1.02)$.

2. (a) The steady state temperature function $T(x, y)$ for a thin flat plate satisfies the equation

$$T_{xx} + T_{yy} = 0.$$

Does the function

$$T(x, y) = \ln(x^2 + y^2)$$

satisfy the equation above? Show your work.

(b) Given

$$f(x, y, z) = z \sin x + \frac{\cos y \ln(z + y)}{z},$$

compute the partial derivative $\dfrac{\partial}{\partial x}\left(\dfrac{\partial f}{\partial z}\right)$.

3. (a) Compute the determinant of the matrix

$$\begin{pmatrix} 5 & 11 \\ -2 & 3 \end{pmatrix}.$$

(b) What are the values of the parameter a for which the matrix

$$M = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & a+2 \\ 1 & 4-a & 5 \end{pmatrix}$$

is invertible?

4. Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 3 & 3 & 3 \end{pmatrix}.$$

5. (a) The linear operator $T : \mathbb{R}^2 \to \mathbb{R}^2$ is defined by

$$T(x_1, x_2) = (3x_1 + 4x_2, x_1 + 3x_2).$$

Find a basis of $\mathbb{R}^2$ such that the matrix of T with respect to that basis is diagonal.

(b) Are there real numbers a and b such that the matrices

$$A = \begin{bmatrix} 0 & 0 & a \\ 1 & 0 & b \\ 0 & 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

are similar (i.e. $A = CBC^{-1}$)? If yes, find a and b. If no, explain why not.

6. Determine the definiteness (positive/negative definite or semidefinite, indefinite) of the following quadratic forms. Explain your answers.

(a) The quadratic form in two variables,

$$Q(x.y) = x^2 + 6xy + 2y^2.$$

(b) The quadratic form associated to the matrix

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}.$$

7. Determine the matrix associated to the following linear transformations. Express your answers in terms of the standard basis.

(a) The transformation $T(x_1, x_2) = (7x_1 - x_2, 8x_1 + 3x_2)$ where $(x_1, x_2) \in \mathbb{R}^2$.

(b) Rotation in $\mathbb{R}^2$ by an angle of $\pi/3$ radians.

(c) Projection of vectors in $\mathbb{R}^3$ onto the line L given by the points

$$\left\{ \begin{bmatrix} 3t \\ -2t \\ t \end{bmatrix} \middle| \, t \in \mathbb{R} \right\}.$$

(d) Reflection in $\mathbb{R}^3$ in the plane given by

$$3y - z = 0.$$

8. Let V be the 2-dimensional subspace of $\mathbb{R}^3$ with basis

$$\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 9 \end{bmatrix} \right\}.$$

(a) Given the vector in $\mathcal{B}$-coordinates

$$\mathbf{v} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}_{\mathcal{B}},$$

express $\mathbf{v}$ in standard coordinates.

(b) Express the vector

$$\mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

written in standard coordinates in terms of the $\mathcal{B}$-coordinates.

9. Let A and B be two invertible $n \times n$ matrices. Show that AB and BA have the same characteristic polynomial.