MATH 51 MIDTERM

October 20, 2005

Name: ______________________________

Numeric Student ID: ____________________

Instructor's Name: _____________________

I agree to abide by the terms of the honor code:

Signature: ___________________________

Instructions: Print your name, student ID number and instructor's name in the space provided. During the test you may not use notes, books or calculators. Read each question carefully and **show all your work**; full credit cannot be obtained without sufficient justification for your answer unless explicitly stated otherwise. Underline your final answer to each question. There are 8 questions. You have 90 minutes to do all the problems.

Question	Score	Maximum
1		10
2		15
3		10
4		10
5		10
6		10
7		10
8		10
Total		85

1. Solve the following system of equations in the variables x, y, z, w:

$$\begin{aligned} x - y - z + w &= 5 \\ y - z + 2w &= 8 \\ 2x - y - 3z + 4w &= 18 \end{aligned}$$

   If a solution exists, express your answer in parametric form.

2. Let A be the matrix

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 4 \\ 1 & 2 & 1 & 1 & 6 \\ 0 & 1 & 1 & 1 & 3 \\ 2 & 2 & 0 & 1 & 7 \end{pmatrix}.$$

   (a) Find a basis for the nullspace of A.

   (b) Find a basis for the column space of A.

   (c) Using your work from the previous page, what is the set of all solutions to the equation

$$A\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 2 \end{pmatrix}, \text{ where } \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}?$$

   State your answer in parametric form.

3. Let P_1 be the plane described by normal vector $(1, 1, 1)$ and containing the point $(0, 0, 1)$. Let P_2 be the plane described by the equation $x + 2y + 3z = 0$.

   (a) Write P_1 as an equation of the form $ax + by + cz = d$.

   (b) What is the set of points in the intersection of P_1 and P_2? If there are points in the intersection, express them in parametric form.

4. The coordinates of three points P, Q and R are $(1, 1, 1)$, $(2, 1, 0)$ and $(3, 2, 3)$ respectively.

   (a) Show that the vectors $\vec{PQ}$ and $\vec{PR}$ are perpendicular.

   (b) Determine the area of the triangle P, Q and R.

5. For each of the following sets, determine whether or not the set is a subspace. For this question only, you do not need to show your work; simply write SUBSPACE or NOT SUBSPACE.

(a) The set $\left\{ \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \text{ in } \mathbb{R}^n : x_1 + x_2 + \cdots + x_n = 1 \right\}$.

(b) The set $\left\{ \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \text{ in } \mathbb{R}^n : x_i \geq 0 \text{ for } 1 \leq i \leq n \right\}$.

(c) The nullspace $N(A)$, where

$$A = \begin{pmatrix} 3 & 0 & 1 \\ 1 & 6 & -17 \end{pmatrix}.$$

(d) The set $\left\{ \begin{pmatrix} x \\ y \end{pmatrix} : y = x^2 \right\}$.

(e) The set $\left\{ \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \text{ in } \mathbb{R}^n : x_1 + x_2 + \cdots + x_n = 0 \right\}$.

6. Let $\{u, v, w\}$ be a linearly independent set of vectors. Show that the set

$$\{u, u + 2v, u + 2v + 3w\}$$

is a linearly independent set of vectors, as well.

7. Let

$$A = \begin{pmatrix} 5 & -2 & 0 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}, \text{ and } \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

Find all solutions to the equation $A\mathbf{x} = 3\mathbf{x}$.

8. Let V be the set of all points P in $\mathbb{R}^4$ such that the distance from P to each one of the points $(1, 2, 3, 4)$, $(2, 3, 4, 1)$, and $(3, 4, 2, 1)$ are all equal. Show that V is a linear subspace of $\mathbb{R}^4$ and compute its dimension.

Note: The distance between two points $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n)$ in $\mathbb{R}^n$ is by definition:

$$\text{dist}(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \ldots + (x_n - y_n)^2}.$$