MATH 51 MIDTERM 2

February 26, 2004

1. Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix}.$$

2. Suppose A and B are 3x3 matrices, and that $\det(A) = 5$ and $\det(B) = -2$.

(a) Find $\det(AB)$.

(b) Find $\det(A^{-1})$.

(c) Find $\det(2A)$.

3. Let

$$A = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}.$$

(a) Find the eigenvalues of A.

(b) Find the eigenvalues of A^{10}.

(c) The matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & -2 \\ 1 & -2 & 3 \end{bmatrix}$$

has the number 3 as one of its eigenvalues. Find an eigenvector $\mathbf{v}$ that has 3 as its associated eigenvalue.

4. Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation defined by:

$$T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + y \\ -2x + 4y \end{bmatrix}.$$

(a). Find the matrix A that represents the linear transformation T with respect to the standard basis $\mathcal{S} = \{\mathbf{e}_1, \mathbf{e}_2\}$.

(b). Consider the basis $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$ given by:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} 3 \\ 7 \end{bmatrix}.$$

Find the change of basis matrix C for the basis $\mathcal{B}$. That is, find the matrix C such that $\mathbf{v} = C[\mathbf{v}]_{\mathcal{B}}$ for all vectors $\mathbf{v}$.

(c). Find the matrix B that represents the linear transformation T with respect to the basis $\mathcal{B}$.

5. Find each of the following limits, or else explain clearly why the limit does not exist.

(a). $\lim_{(x,y)\to(2,3)} \dfrac{e^{xy}\sin y}{2x+y}$.

(b). $\lim_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$.

(c). $\lim_{(x,y)\to(0,0)} \dfrac{xy\sin x}{x^2+2y^2}$.

6(a). Find a matrix A such that

$$A\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}1\\2\end{bmatrix}, \qquad A\begin{bmatrix}1\\1\\0\end{bmatrix} = \begin{bmatrix}4\\3\end{bmatrix}, \qquad A\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}7\\7\end{bmatrix}.$$

(b). Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be reflection across the line $y = 3x$. Find the matrix for T (with respect to the standard basis of $\mathbf{R}^2$.)

7. A particle moves through space with velocity given by $\mathbf{v}(t) = (\sin t, 2t, 1)$. At time $t = 0$, the particle's position is $(2, 3, 4)$.

(a) Find the particle's speed at time t.

(b) Find the particle's acceleration vector at time t.

(c) Find the particle's position $\mathbf{x}(t)$ at time t.

8. Calculate the following partial derivatives:

(a) $\dfrac{\partial}{\partial x}(xy^2z + y^2\sin x + yz^5)$

(b) $\dfrac{\partial}{\partial y}\sin(xyz + x^2)$

(c) $\dfrac{\partial f}{\partial x}(2,3)$ where $f(x,y) = x^2 + xy + y^2$.

(d) $\dfrac{\partial^3 u}{\partial x\partial y\partial z}$ where $u(x,y,z) = xy^2z^3$.

9(a). Find the determinant of the matrix

$$A = \begin{bmatrix}1 & 1 & 1\\0 & 2 & 1\\c & 0 & 3\end{bmatrix}.$$

(Your answer should be an expression involving c.)

9(b). Let U be the ball of radius 1 centered at the origin. For which values of c will U and $A(U)$ have the same volume?

10. Suppose a particle moves with constant speed 5. Prove that at each time t, the particle's velocity and acceleration vectors are perpendicular to each other.