MATH 51 FINAL EXAM SOLUTIONS

Professors Clingher, Munson, and White

March 15, 2004

1 Consider the matrices

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 2 \\ 2 & 2 & 0 & 1 & 6 \\ 0 & 1 & -1 & 1 & 3 \\ -1 & -2 & 1 & 1 & -1 \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The matrix R is the row reduced echelon form of A. (You do not need to check this.)

1(a). Find a basis for the column space of A. Solution: R has pivots in columns 1, 2 and 4, so the corresponding columns of A form the basis:

$$\begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 2 \\ 1 \\ -2 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

1(b). Find a basis for the column space of R. Solution: R is its own rref, so again the pivot columns of R (that is, the first, second, and fourth columns) form the basis: $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$.

1(c). Find a basis for the nullspace of A. The nullspace of A consists of all solutions of $A\mathbf{x} = 0$, or, equivalently, of $R\mathbf{x} = 0$. From $R\mathbf{x} = 0$, we see that the free variables are x_3 and x_5 and that

$$\mathbf{x} = \begin{bmatrix} -x_3 - x_5 \\ x_3 - x_5 \\ x_3 \\ -2x_5 \\ x_5 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -1 \\ -1 \\ 0 \\ -2 \\ 1 \end{bmatrix}.$$

Thus $\begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ -1 \\ 0 \\ -2 \\ 1 \end{bmatrix}$ form a basis for $N(A)$.

2. Find all solutions of

$$\begin{aligned} x_1 + 2x_2 + x_3 + x_4 &= 7 \\ x_1 + 2x_2 + 2x_3 - x_4 &= 12 \\ 2x_1 + 4x_2 \qquad\quad + 6x_4 &= 4. \end{aligned}$$

Solution: see example 6.4 on page 44 of the Levandosky text. The rref is

$$\left[\begin{array}{cccc:c} 1 & 2 & 0 & 3 & 2 \\ 0 & 0 & 1 & -2 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

from which we get the solution:

$$\mathbf{x} = \begin{bmatrix} 2 \\ 0 \\ 5 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -3 \\ 0 \\ 2 \\ 1 \end{bmatrix}.$$

3(a). Find all eigenvalues of the matrix $A = \begin{bmatrix} 5 & 0 & 0 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$. Solution:

$$\begin{aligned} 0 = \det(\lambda I - A) &= \begin{vmatrix} \lambda - 5 & 0 & 0 \\ -1 & \lambda - 2 & -1 \\ -1 & -1 & \lambda - 2 \end{vmatrix} \\ &= (\lambda - 5) \begin{vmatrix} \lambda - 2 & -1 \\ -1 & \lambda - 2 \end{vmatrix} \\ &= (\lambda - 5)((\lambda - 2)^2 - (-1)^2) = (\lambda - 5)(\lambda - 3)(\lambda - 1) \end{aligned}$$

so the eigenvalues are 5, 3, and 1.

3(b). The matrix $M = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ has $\lambda = 2$ as one of its eigenvalues. (You need not check this.) Let V be the eigenspace corresponding to this eigenvalue. (In other words, V consists of all eigenvectors with eigenvalue 2 together with the origin.) Find a basis for V.

Solution: $2I - M = \begin{bmatrix} -3 & 6 & 6 \\ 1 & -2 & -2 \\ -3 & 6 & 6 \end{bmatrix}$. Note that V is the nullspace of $2I - M$. We find this nullspace by solving $(2I - M)\mathbf{v} = 0$. Solving by Gaussian elimination, we get the single equation

$$x - 2y - 2z = 0$$

so y and z are free and

$$\mathbf{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2y + 2z \\ y \\ z \end{bmatrix} = y \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}.$$

Thus $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ form a basis for V.

4. The velocity of a certain spaceship at time t is given by $\mathbf{v}(t) = (3t^2, e^{t-1}, 6t)$. At time $t = 1$, its position is $(0, 0, 7)$.

(a) Find the speed at time t. **Solution:**

$$\|\mathbf{v}(t)\| = \sqrt{(3t^2)^2 + (e^{t-1})^2 + (6t)^2} = \sqrt{9t^4 + e^{2t-2} + 36t^2}.$$

(b) Find the acceleration at time t. **Solution:**

$$\mathbf{v}'(t) = (6t, e^{t-1}, 6).$$

(c) At time $t = 1$, the spaceship's rearview mirror breaks off and then continues to move with constant velocity (i.e., with the same constant velocity it had at time 1). Find the position of the mirror at time t (for $t \geq 1$).

Solution: Let $\mathbf{u}(t)$ be the mirror's position at time t. Then for $t \geq 1$:

$$(*) \qquad \mathbf{u}'(t) = \mathbf{v}(1) = (3(1)^2, e^{1-1}, 6(1)) = (3, 1, 6).$$

Here we can use definite or indefinite integration. With indefinite integration:

$$(\dagger) \qquad \mathbf{u}(t) = \int (3, 1, 6)\, dt = (3t, t, 6t) + \mathbf{C}.$$

Plugging in $t = 1$ gives $(0, 0, 7) = (3, 1, 6) + \mathbf{C}$, so $=(-3, -1, 1)$. Thus

$$\boxed{\mathbf{u}(t) = (-3, -1, 1) + t(3, 1, 6).}$$

With definite integration: integrate (*) from $t = 1$ to T:

$$\begin{aligned} \mathbf{u}(T) &= \mathbf{u}(1) + \int_{t=1}^{T} \mathbf{u}'(t)\, dt \\ &= (0, 0, 7) + \int_{1}^{T} (3, 1, 6)\, dt \\ &= (0, 0, 7) + (t - 1)(3, 1, 6) \\ &= \boxed{(3t - 3, t - 1, 6t + 1)}. \end{aligned}$$

5. Find each of the following limits, or else explain clearly why the limit does not exist.

(a). $\lim_{(x,y)\to(0,9)} \dfrac{xy}{x^2 + y^2 + 2}$. **Solution**: The numerator and denominator are continuous, and the denominator is never 0, so the quotient is continuous everywhere. Thus we get the limit by putting $x = 0$ and $y = 9$: $\dfrac{(0)(9)}{0^2 + 9^2 + 2} = 0$.

(b). $\lim_{(x,y)\to(0,0)} \dfrac{(x - y)^2}{x^2 + y^2}$. **Solution:** In polar coordinates (i.e., $x = r\cos\theta$ and $y = r\sin\theta$), the expression becomes: $\dfrac{(r\cos\theta - r\sin\theta)^2}{r^2} = (\cos\theta - \sin\theta)^2$. If we let $r \to 0$ keeping θ fixed, this expression tends to $(\cos\theta - \sin\theta)^2$. Thus the "limit" depends on the direction from which we approach $(0, 0)$, so the limit does not actually exist.

(c). $\lim_{(x,y)\to(0,0)} \dfrac{x^3 - y^3}{x^2 + y^2}$. **Solution:** In polar coordinates, the expression is

$$\frac{r^3 \cos^3\theta - r^3 \sin^3\theta}{r^2} = r(\sin^3\theta - \cos^3\theta)$$

which goes to 0 as $r \to 0$. Hence the limit exists and is 0.

6. Find the matrix derivative (i.e., the Jacobian matrix) $DF(x, y, z)$ where

$$F(x, y, z) = \begin{bmatrix} x + y^2 + z^3 \\ e^y + y \sin z \end{bmatrix}.$$

Solution: $\begin{bmatrix} 1 & 2y & 3z^2 \\ 0 & e^y + \sin z & y \cos z \end{bmatrix}$

7. Find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & 4 \end{bmatrix}$.

Solution: Row reduce $[A : I]$. An easy way to row reduce it is: subtract row 4 from each of the first 3 rows, then subtract row 3 (of the resulting matrix) from each the first two rows, then subtract row 2 from row 1:

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 4 & 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 4 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cccc|cccc} 1 & 2 & 3 & 0 & 1 & 0 & 0 & -1 \\ 0 & 2 & 3 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 3 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1/2 & -1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/3 & -1/3 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1/4 \end{array}\right]$$

(where in the last step we divided row 4 by 4, row 3 by 3, row 2 by 2.)

Hence $A^{-1} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1/2 & -1/2 & 0 \\ 0 & 0 & 1/3 & -1/3 \\ 0 & 0 & 0 & 1/4 \end{bmatrix}$.

8. Find the point or points (x, y) at which the function

$$f(x,y) = \frac{x^4}{4} - xy + \frac{y^2}{2}$$

is a minimum. (You may assume that the minimum exists. Note the different exponents.)

Solution: At the minimum, $0 = f_x = x^3 - y$ and $0 = f_y = -x + y$. So (from the second equation) $y = x$. Combining with the first gives $0 = x^3 - x = x(x^2 - 1) = x(x-1)(x+1)$, so $x = -1$, 0, or 1. Since $y = x$, the minimum must be at $(-1, -1)$, $(0, 0)$, or $(1, 1)$. The corresponding values of f are $-1/4$, 0, and $-1/4$. Thus the minimum occurs at $(-1, -1)$ and at $(1, 1)$.

9(a). Consider the linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ given by first reflecting across the line $y = x$, and then rotating counterclockwise around the origin by an angle of $\pi/2$. Find the matrix A for this linear transformation (with respect to the standard basis $\{\mathbf{e}_1, \mathbf{e}_2\}$ of $\mathbf{R}^2$).

Solution: The vector $\mathbf{e}_1$ gets reflected to $\mathbf{e}_2$, which is then rotated to $-\mathbf{e}_1$. So the first column of A is $-\mathbf{e}_1$. The vector $\mathbf{e}_2$ gets reflected to $\mathbf{e}_1$, which is then rotated to $\mathbf{e}_2$. So the second column is $\mathbf{e}_2$. Thus

$$\boxed{A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}.}$$

9(b). Find the matrix B for T with respect to the basis $\mathcal{B}$ consisting of $\mathbf{v}_1 = (1, 1)$ and $\mathbf{v}_2 = (-1, 1)$. (Note: it is possible to do part (b) directly, without doing any calculations and without using the answer to part (a).)

Solution: The vector $\mathbf{v}_1$ gets reflected to itself, then rotated to $\mathbf{v}_2 = 0\mathbf{v}_1 + 1\mathbf{v}_2$. Thus the first column of B is $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$. The vector $\mathbf{v}_2$ gets reflected to $-\mathbf{v}_2$, which is then rotated to $\mathbf{v}_1 = 1\mathbf{v}_1 + 0\mathbf{v}_2$. So the second column of B is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Thus

$$\boxed{B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.}$$

Another approach: The change of basis matrix is

$$C = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix},$$

so

$$B = C^{-1}AC = \begin{bmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

10. The temperature at point (x, y) on the floor of a room is given by $f(x, y) = xy^2$.

(a) A tweetle beetle crawls on the floor. At time $t = 2$, he is at the point $(5, 1)$ and his velocity is $(2, -1)$. Find the rate of change of his temperature at time $t = 2$.

Solution:

$$\begin{aligned} \frac{df}{dt} &= \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} \\ &= y^2\frac{dx}{dt} + 2xy\frac{dy}{dt} = (1^2)(2) + (2 \cdot 5 \cdot 1)(-1) = \boxed{-8}. \end{aligned}$$

(b) Another tweetle beetle is at the point $(1, 3)$, where she finds it uncomfortably cold. In which direction should she start moving in order to warm up as quickly as possible?

Solution: In the direction of $\nabla f = (y^2, 2xy) = \boxed{(1, 10)}$.

(c) A third beetle crawls along the floor keeping his temperature constant. At time $t = 0$, he is at the point $(1, 1)$ and the x-component of his velocity is 7. Find the y-component of his velocity at $t = 0$.

Solution:

$$\begin{aligned} 0 &= \frac{df}{dt} \\ &= \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} \\ &= y^2\frac{dx}{dt} + (2xy)\frac{dy}{dt} \\ &= 1^2 \cdot 7 + (2 \cdot 1 \cdot 1)\frac{dy}{dt} \\ &= 7 + 2\frac{dy}{dt}, \end{aligned}$$

so $\boxed{\frac{dy}{dt} = -7/2}$ at $t = 0$.

11. Find the maximum and the minimum values of $f(x, y, z) = 2x + y + 4z$ on the region $x^2 + y^2/2 + z^2 \leq 22$.

Solution: $\nabla f = (2, 1, 4)$ which is never 0, so no maxima or minima in the interior of the region. For a maximum or a minimum at the boundary: ∇f and

$$\nabla g = (2x, y, 2z)$$

must be dependent. (Here $g(x, y, z) = x^2 + y^2/2 + z^2$.) Since ∇f is never 0, this means

$$\begin{aligned} \nabla g &= k\nabla f \\ (2x, y, 2z) &= k(2, 1, 4) \end{aligned}$$

so

(*) $$(x, y, z) = (k, k, 2k).$$

Plugging this into the constraint equation gives:

$$\begin{aligned} k^2 + k^2/2 + 4k^2 &= 22 \\ (11/2)k^2 &= 22 \\ k^2 &= 4 \end{aligned}$$

so $k = 2$ or $k = -2$. Hence (by (*)) (x, y, z) is either $(2, 2, 4)$ or $(-2, -2, -4)$.

$$f(2, 2, 4) = 2(2) + 2 + 4(4) = 22, \qquad f(-2, -2, -4) = -22$$

so the maximum and minimum values are 22 and -22, respectively.

12(a) Suppose that $\mathbf{x}$ and $\mathbf{y}$ are vectors in $\mathbf{R}^n$ with $\|\mathbf{x}\| = 2$ and $\|\mathbf{y}\| = 1$. Suppose also that the angle between $\mathbf{x}$ and $\mathbf{y}$ is $\theta = \arccos(1/4)$. Prove that the vectors $\mathbf{x} - 3\mathbf{y}$ and $\mathbf{x} + \mathbf{y}$ are orthogonal.

Solution: $$\mathbf{x} \cdot \mathbf{y} = \|\mathbf{x}\| \, \|\mathbf{y}\| \, \cos\theta = (2)(1)(1/4) = 1/2,$$

so

$$(\mathbf{x} - 3\mathbf{y}) \cdot (\mathbf{x} + \mathbf{y}) = \|\mathbf{x}\|^2 - 2\mathbf{x} \cdot \mathbf{y} - 3\|\mathbf{y}\|^2 = 2^2 - 2(1/2) - 3(1^2) = 0.$$

Thus the vectors are perpendicular.

12(b). Prove there is no matrix A (with real entries) such that $A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$.

(Hint: use determinants.)

Solution: If the equation were true, then

$$\det(A^2) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{vmatrix} = -4.$$

But $\det(A^2) = \det(AA) = (\det A)(\det A) = (\det A)^2$. So $\det A$ would have to be a (real) number whose square is -4. But that's impossible.

13(a). Find a normal vector to S at $\mathbf{p} = (1, 2, 3)$, where S is the surface

$$x + y + z + xyz = 12.$$

Solution: The surface is a level set of $f(x, y, z) = x + y + z + xyz$, so

$$\nabla f = (1 + yz, 1 + xz, 1 + xy)$$

is normal at (x, y, z). Plugging in $(x, y, z) = (1, 2, 3)$ gives a normal vector at $\mathbf{p}$:

$$\boxed{\mathbf{N} = \nabla f(\mathbf{p}) = (7, 4, 3).}$$

13(b). Find an equation for the tangent plane to S at $\mathbf{p}$ (where S and $\mathbf{p}$ are in part (a).)

Solution: The equation is $\mathbf{N} \cdot (\mathbf{v} - \mathbf{p}) = 0$ (where $\mathbf{v} = (x, y, z)$), or

$$\boxed{(7,4,3) \cdot ((x,y,z) - (1,2,3)) = 0}$$

or

$$\boxed{(7,4,3) \cdot ((x-1, y-2, z-3) = 0}$$

or

$$\boxed{7(x-1) + 4(y-2) + 3(z-3) = 0.}$$

14(a). Consider a function $G : \mathbf{R}^2 \to \mathbf{R}^2$ such that

$$G(5,2) = \begin{bmatrix} 17 \\ 13 \end{bmatrix} \qquad \text{and} \qquad DG(x,y) = \begin{bmatrix} (xy-1)^{1/3} & (xy-1)^{1/3} \\ \sin(y-2) & x\cos(y-2) \end{bmatrix}.$$

Estimate $G(5.02, 2.001)$. (Hint: use linear approximation. Here DG denotes the Jacobian matrix, i.e., the matrix derivative.))

Solution:

$$\begin{aligned} G(5.02, 2.001) &\cong G(5,2) + DG(5,2) \begin{bmatrix} .02 \\ .001 \end{bmatrix} \\ &= \begin{bmatrix} 17 \\ 13 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} .02 \\ .001 \end{bmatrix} = \begin{bmatrix} 17 \\ 13 \end{bmatrix} + \begin{bmatrix} .063 \\ .005 \end{bmatrix} = \begin{bmatrix} 17.063 \\ 13.005 \end{bmatrix} \end{aligned}$$

14(b). Consider a function $H : \mathbf{R}^2 \to \mathbf{R}^2$ such that:

$$H(0,0) = (0,0) \qquad \text{and} \qquad DH(0,0) = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}.$$

Suppose (x, y) is a point near $(0, 0)$ such that $H(x, y) = (.07, .06)$. Estimate x and y. (Hint: use linear approximation. Here DH denotes the Jacobian matrix, i.e., the matrix derivative.)

Solution: $\begin{bmatrix} .07 \\ .06 \end{bmatrix} = H(x,y) \cong H(0,0) + DH(0,0) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+y \\ 2x \end{bmatrix}$

so

$$\begin{aligned} x + y &\cong .07 \\ 2x &\cong .06 \end{aligned}$$

solving (just as if we had equality instead of $\cong$) gives $\boxed{x \cong .03}$ and $\boxed{y \cong .04}$.

15. Let P be the plane given by

$$x_1 + x_2 + x_3 = 0.$$

Let A be the matrix that represents projection onto P. Thus if $\mathbf{x}$ is point in $\mathbf{R}^3$, then $A\mathbf{x}$ is in P and $\mathbf{x} - A\mathbf{x}$ is perpendicular to P. Another way of saying this: $A\mathbf{x}$ is the point in P nearest to $\mathbf{x}$. (Note: it is possible to do both parts of this problem without actually finding A.)

(a) Find a basis for the null space of A.

Solution: The nullspace is the line L through the origin perpendicular to P. Since P is a level set of the function $x_1 + x_2 + x_3$, the gradient $\nabla f = (1, 1, 1)$ gives is normal to P. That is, $(1, 1, 1)$ is a nonzero vector in the line L. Thus $(1, 1, 1)$ forms a basis for L, i.e., for $N(A)$.

(b) Find a basis for the column space of A.

Solution: The column space $C(A)$ of A is the image of A, which is just the plane P. Thus we need a basis for P. Note that

$$x_1 + x_2 + x_3 = 0$$

is already in row reduced echelon form, so x_2 and x_3 are free variables and the general solution is

$$\mathbf{x} = \begin{bmatrix} -x_2 - x_3 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} x_3.$$

So $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ form a basis for $C(A)$.