



5. Find the maximum and minimum of  $f(x, y) = xy$  on the region where

$$\frac{9}{2}x^2 + \frac{1}{2}y^2 \leq 36.$$

Draw a diagram of the region and indicate clearly the point(s) where the maximum occurs, the point(s) where the minimum occurs, and any other points you had to test. (The following page is blank in case you need more space.)

5. (Continued)

6(a). Find the partial derivative  $g_{xy}$ , where  $g(x, y) = x^3y + 7xy^2$ .

6(b). Find the matrix derivative (i.e., the Jacobian matrix)  $DF(x, y)$  where

$$F(x, y) = \begin{bmatrix} x \sin y \\ y^2 \\ 2x + 3y \end{bmatrix}.$$

7. A function  $z = z(x, y)$  satisfies the equation

$$2x + yz + z^3 = 9.$$

Find  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$  at the point  $(x, y, z) = (3, 2, 1)$ .

8. Waves move across a lake so that the surface of the water at time  $t$  is given by:

$$z = \sin((0.1)x + (0.2)y - t).$$

An insect skims across the lake's surface. At time  $t = 0$ , the insect is at the origin and the  $x$  and  $y$  components of its velocity are 3 and 7, respectively. Find the  $z$ -component of its velocity at time 0.

9. Let  $f(x, y, z)$  denote the temperature at point  $(x, y, z)$  (where temperature is in degrees celsius and distances are in centimeters.) Suppose that  $f(0, 0, 0) = 10$  and that  $\nabla f(0, 0, 0) = (2, 3, 1)$ .

9(a). Estimate the temperature at the point  $(0.1, 0.1, 0.4)$ .

9(b). Let  $(a, b, c)$  be the point with temperature 10.28 that is closest to the origin. Using the the gradient, estimate  $(a, b, c)$ .

[Hint: if you set off from the origin at a given speed  $s$ , in which direction should you go if you wish to reach the level set  $f(x, y, z) = 10.28$  as quickly as possible?]

10. Find the inverse of the matrix  $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ .

11. Let  $T : \mathbf{R}^3 \rightarrow \mathbf{R}^3$  be rotation by  $180^\circ$  about the  $x$ -axis followed by reflection in the plane  $x = y$ . Find the matrix for  $T$ .

12(a). Consider the line  $L$  that passes through the point  $\mathbf{p} = (5, 1, 0)$  and that is perpendicular to the plane  $7x - 2y + z = 14$ . Find a parametric representation for  $L$ .

**12(b).** Consider the triangle in  $\mathbf{R}^4$  with corners at  $A = (1, 1, 1, 1)$ ,  $B = (1, 4, 5, 1)$ , and  $C = (1, 5, 4, 1)$ . Find the length of the side  $AB$ .

**12(c).** Find the cosine of the angle at vertex  $A$  of the triangle  $ABC$  (where  $A$ ,  $B$  and  $C$  are as in part (b).)

**13.** Consider the surface  $S$  given by the equation

$$xyz = x + y + z.$$

Find an equation for the tangent plane to  $S$  at the point  $\mathbf{p} = (1, 2, 3)$ .

**14(a).** Suppose that  $\mathbf{v}_1$ ,  $\mathbf{v}_2$ , and  $\mathbf{v}_3$  are linearly dependent vectors in  $\mathbf{R}^n$ . Let  $T : \mathbf{R}^n \rightarrow \mathbf{R}^m$  be a linear map. Prove that the vectors  $T(\mathbf{v}_1)$ ,  $T(\mathbf{v}_2)$ , and  $T(\mathbf{v}_3)$  are also linearly dependent.

**14(b).** Suppose a particle moves in  $\mathbf{R}^3$  with constant speed. Prove that the acceleration vector and the velocity vector must always be orthogonal to each other.

[Note: the velocity of the particle need not be constant.]