EXAM I

Math 51, Spring 2001.

You have 2 hours.

No notes, no books.

YOU MUST SHOW ALL WORK TO RECEIVE CREDIT

Good luck!

Name ________________________________

ID number______________________

1. __________ (/20 points)

2. __________ (/20 points)

3. __________ (/20 points)

4. __________ (/20 points)

5. __________ (/20 points)

Bonus __________ (/10 points)

Total __________ (/100 points)

"On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: ______________________

Circle your TA's name:

Kuan Ju Liu (2 and 6)

Robert Sussland (3 and 7)

Hunter Tart (4 and 8)

Alex Meadows (10)

Dana Rowland (11)

Circle your section meeting time:

11:00am 1:15pm 7pm

1. (a) Define what it means for a set of vectors $\{\overrightarrow{v}_1, \ldots, \overrightarrow{v}_k\}$ to be "linearly dependent".

(b) Find a linear dependence of the three vectors below, or prove that they are independent.

$$\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 5 \\ 7 \end{bmatrix}$$

(c) Prove that three nonzero vectors in $\mathbb{R}^2$ must be dependent.

2. Suppose that V and W are vector subspaces of $\mathbb{R}^n$.

   (a) Show that the intersection $V \bigcap W$ (in other words, the set of all vectors that are in both V and W) is also a vector subspace of $\mathbb{R}^n$.

(b) Prove that the union $V \bigcup W$ (in other words, the set of all vectors that are in either V or W) is a vector space – or find a counterexample to refute this statement.

3. (a) Find a parametric representation of the set of solutions to the system of equations below.

$$\begin{array}{rcrcrcr} 3x & + & 2y & + & z & = & -1 \\ -2x & & & + & 3z & = & 2 \end{array}$$

(b) Given this result and your knowledge of the relationship between solution sets and the null space, find a basis for the null space of the matrix

$$\begin{pmatrix} 3 & 2 & 1 \\ -2 & 0 & 3 \end{pmatrix}$$

(c) Use the cross product to find a basis for the null space of the matrix below (DO NOT row reduce as in parts (a) and (b); and make sure to **explain why your calculation is valid**.)

$$\begin{pmatrix} 2 & 4 & 0 \\ 1 & -1 & 2 \end{pmatrix}$$

4. (a) Suppose that the vectors $\overrightarrow{v}$, $\overrightarrow{w}$ and $\overrightarrow{x}$ are mutually perpendicular. Use dot products to find

$$\|\overrightarrow{v} + 3\overrightarrow{w} + 2\overrightarrow{x}\|$$

in terms of the lengths of $\overrightarrow{v}$, $\overrightarrow{w}$ and $\overrightarrow{x}$.

(b) Show that if a vector $\overrightarrow{v}$ is perpendicular to the row vectors $\overrightarrow{r_1}, \ldots \overrightarrow{r_m}$ of a matrix A, then the vector $\overrightarrow{v}$ must be in the null space of A.

5. The matrix A below has the given reduced row echelon form (You do not need to verify this).

$$A = \begin{pmatrix} 3 & 4 & 0 & 7 \\ 1 & -5 & 2 & -2 \\ -1 & 4 & 0 & 3 \\ 1 & -1 & 2 & 2 \end{pmatrix} \qquad \text{rref}(A) = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Using this information, write down bases for the row space, column space, and null space of A.

Bonus Question: Suppose we have three matrices A, B_1, and B_2, with the following properties:

1. A is 4x4, B_1 is 4x3, and B_2 is 3x4
2. $A\overrightarrow{v} = B_1(B_2\overrightarrow{v})$ for all vectors $\overrightarrow{v}$ in $\mathbb{R}^4$

Show that there must exist a non-zero vector in $N(A)$, and that there must also exist a vector in $\mathbb{R}^4$ which is not in $C(A)$.