MATH 51 MIDTERM 2

November 16, 2000

	Brumfiel	Hutchings	Levandosky	Staffilani	White
11:00	01	05	09	13	17
1:15	03	07	11	15	19

Name: ______________________________

Student ID: ___________________________

Signature: ____________________________

Instructions: Print your name and student ID number and write your signature to indicate that you accept the honor code. Circle the number of the section for which you are registered on Infopier. During the test, you may not use notes, books, or calculators. Read each question carefully, and show all your work. Put a box around your final answer to each question. You have 90 minutes to do all the problems.

Question	Score
1	
2	
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6	
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9	
10	
Total	

1. (a) Compute the inverse of the matrix

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 2 & 4 \\ 0 & 0 & -1 \end{bmatrix}$$

(b) For which value(s) of x is the matrix below **not** invertible? Explain your answer.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 5 & x & 6 \end{bmatrix}$$

2. (a) Suppose

$$A = \begin{bmatrix} 2/3 & -1/3 & 2/3 \\ 2/3 & 2/3 & -1/3 \\ -1/3 & 2/3 & 2/3 \end{bmatrix}$$

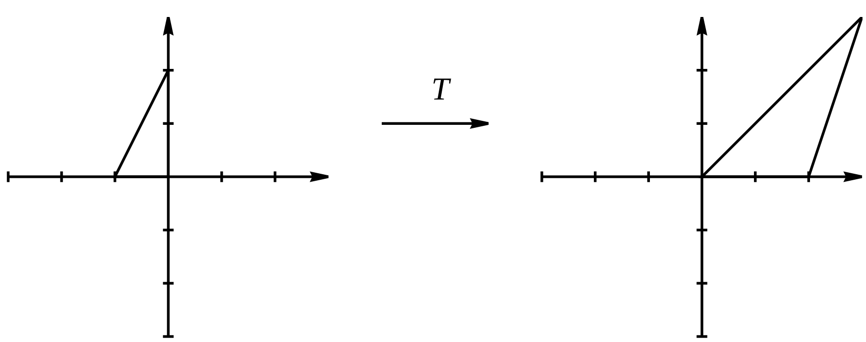
is the matrix of a linear transformation which is geometrically a 60 degree rotation about a line L in $\mathbf{R}^3$. Find the matrix of a 120 degree rotation about L. Hint: Think about composition.

(b) Let

$$B = \begin{bmatrix} 2 & 2 & 3 & 5 \\ 4 & 3 & 2 & 1 \\ -1 & 2 & -1 & 2 \\ 9 & 8 & 5 & 8 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} 6 \\ 4 \\ -2 \\ 10 \end{bmatrix}$$

Compute $B^{-1}\mathbf{v}$. Hint: You do not need to compute B^{-1}. Compare $\mathbf{v}$ with the columns of B.

3. Let Δ_1 be the triangle with vertices $(0,0)$, $(-1,0)$ and $(0,2)$ and let Δ_2 be the triangle with vertices $(0,0)$, $(2,0)$ and $(3,3)$. Suppose $T : \mathbf{R}^2 \to \mathbf{R}^2$ is a linear transformation such that $T(\Delta_1) = \Delta_2$.



(a) There are exactly two such linear transformations. Find the matrix for one of them.

(b) Let E represent the region bounded by the ellipse

$$\frac{x^2}{4} + \frac{y^2}{25} = 1$$

The area of E is 10π. Find the area of $T(E)$. Note: The answer is the same for both linear transformations T which satisfy $T(\Delta_1) = \Delta_2$.

4. Let

$$A = \begin{bmatrix} 5 & 1 \\ 1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & -1 & 0 \\ 2 & 1 & 1 \\ 5 & -7 & 6 \end{bmatrix}$$

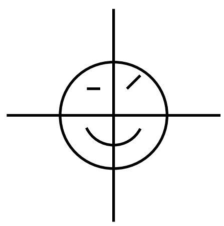
(a) Find the eigenvalues of A.

(b) $\lambda = 3$ is an eigenvalue of B. (You do not need to check this.) Find all eigenvectors of B with eigenvalue 3.

5. Let

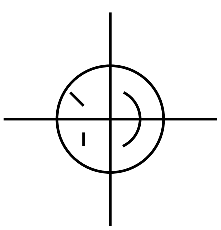
$$A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad D = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad E = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Let S denote the set of points in the face shown below.

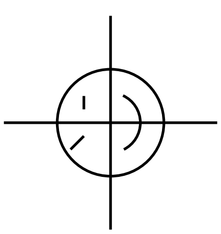


Each figure below is the image of S under the linear transformation corresponding to one of the matrices above. Match each figure with the corresponding matrix.

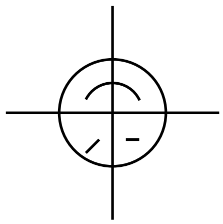
(a)



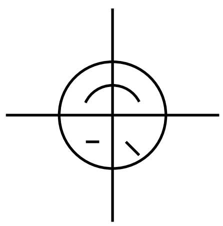
(b)



(c)



(d)



6. (a) Compute the following limit. Explain your answer.

$$\lim_{(x,y,z)\to(2,3,-1)} \frac{xy^2z - 2xyz}{x^2y + xz + y^2z^2}$$

(b) Show that the following limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{2x^2 + y^2}{x^2 + 2y^2}$$

7. Let $f(x, y) = xy + \sin(2x - 4y)$.

(a) Suppose an ant is crawling on a surface whose height in cm at the point (x, y) is given by $f(x, y)$. If the ant is crawling in such a way that its x-coordinate is increasing at $2cm/sec$ and its y-coordinate is increasing at $1cm/sec$, at what rate is its height changing when the (x, y) coordinates of the ant are $(2, 1)$?

(b) Find $\frac{\partial^2 f}{\partial y \partial x}(x, y)$ and $\frac{\partial^2 f}{\partial x^2}(x, y)$.

8. Let $f : D \subset \mathbf{R}^2 \to \mathbf{R}$ be defined by $f(x, y) = \sqrt{xy + y^2}$.

(a) Sketch the domain D of f. Hint: $xy + y^2 = y(x + y)$.

(b) Find $Jf(3, 1)$.

(c) Use the answer to part (b) to find an approximation of $f(3.01, 1.02)$.

9. Define $\mathbf{f} : \mathbf{R}^2 \to \mathbf{R}^3$ and $\mathbf{g} : \mathbf{R}^3 \to \mathbf{R}^2$ by

$$\begin{aligned} \mathbf{f}(x, y) &= (xy, x^2 + y^2, 2x - 2y) \\ \mathbf{g}(x, y, z) &= (x^2 + y^2 + z^2, xyz) \end{aligned}$$

Find the following Jacobian matrices.

(a) $J\mathbf{f}(1, 1)$.

(b) $J\mathbf{g}(1, 2, 0)$.

(c) $J(\mathbf{g} \circ \mathbf{f})(1, 1)$.

10. Each figure below represents the level curves of some function. (The graphs are shown in the usual orientation, with the x-axis horizontal and the y-axis vertical.)

Figure 1

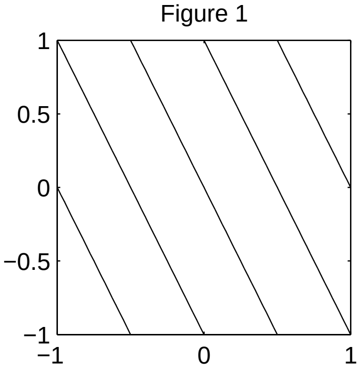


Figure 2

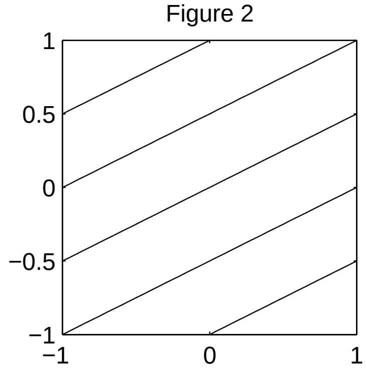


Figure 3

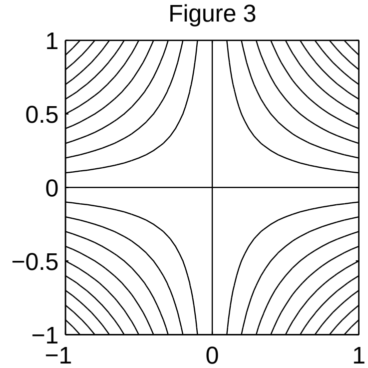


Figure 4

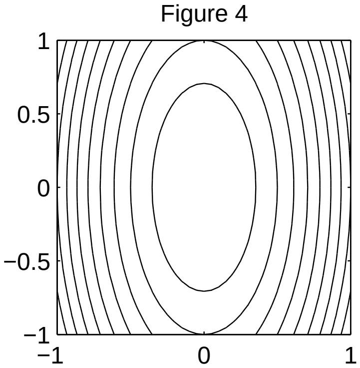


Figure 5

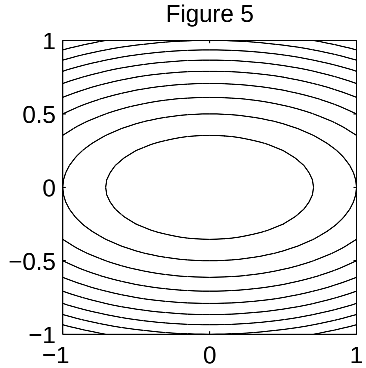
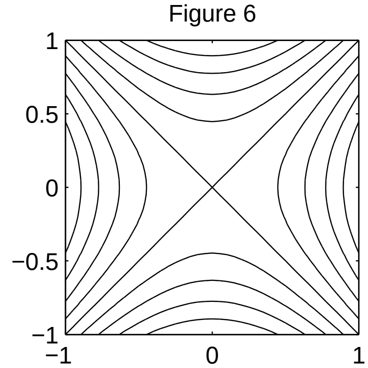


Figure 6



For each function below, indicate which figure represents its level curves.

(a) $x - 2y$

(b) xy

(c) $x^2 + 4y^2$