MATH 51 MIDTERM 1

October 19, 2000

	Brumfiel	Hutchings	Levandosky	Staffilani	White
11:00	01	05	09	13	17
1:15	03	07	11	15	19

Name: ______________________________

Student ID: ______________________________

Signature: ______________________________

Instructions: Print your name and student ID number and write your signature to indicate that you accept the honor code. Circle the number of the section for which you are registered on Infopier. During the test, you may not use notes, books, or calculators. Read each question carefully, and show all your work. Put a box around your final answer to each question. Each of the ten problems is worth 10 points. You have 90 minutes to do all the problems.

Question	Score
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

1. Find all solutions of the following system:

$$\begin{array}{ccccccccc} x_1 & + & x_2 & & & + & x_4 & = & 7 \\ x_1 & + & x_2 & + & x_3 & + & x_4 & = & 10 \\ x_1 & & & + & x_3 & + & x_4 & = & 9 \end{array}$$

2. Let L be the intersection of the two planes

$$x + 2y + 3z = 10 \qquad \text{and} \qquad 4x + 5y + 6z = 28.$$

Find a parametric equation for L.

3. (a) Suppose $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbf{R}^n$ such that $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$ are orthogonal (i.e., perpendicular) to each other. Show that $\|\mathbf{u}\| = \|\mathbf{v}\|$.

   (b) Suppose $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are unit vectors in $\mathbf{R}^n$. (Recall that a unit vector is a vector whose length is 1.) Suppose each vector is orthogonal (i.e., perpendicular) to each of the other two. Show that the two vectors

$$\mathbf{u} - 3\mathbf{v} + 2\mathbf{w} \qquad \text{and} \qquad \mathbf{u} + \mathbf{v} + \mathbf{w}$$

   are orthogonal to each other.

4. Consider the points $A = (1, 1, 1)$, $B = (1, 3, 1)$ and $C = (1, 1, 4)$ in $\mathbf{R}^3$.

   (a) Find the cosine of the angle at B of the triangle ABC.

   (b) Find a parametric equation for the plane through the points A, B, and C.

5. Are the following three vectors in $\mathbf{R}^4$ linearly independent or linearly dependent? Show your work and explain your answer.

$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ -1 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} -1 \\ 2 \\ -2 \\ 4 \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} 3 \\ 2 \\ 8 \\ 2 \end{bmatrix}$$

6. Let

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 3 \\ 1 & 3 & 7 \\ 2 & 2 & 2 \end{bmatrix}.$$

What condition(s) must $\mathbf{b}$ satisfy to be in the column space of A? (Your answer should be one or more equations of the form $?\,b_1 + ?\,b_2 + ?\,b_3 + ?\,b_4 = ?$.)

7. (a) Suppose $\mathbf{u}$ is a vector in $\mathbf{R}^4$. Let V be the set of all vectors in $\mathbf{R}^4$ which are orthogonal (i.e. perpendicular) to $\mathbf{u}$. That is,

$$V = \{\mathbf{x} \in \mathbf{R}^4 \mid \mathbf{x} \cdot \mathbf{u} = \mathbf{0}\}.$$

   Show that V is a subspace of $\mathbf{R}^4$.

(b) Suppose the vector $\mathbf{u}$ in part (a) is

$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}.$$

Find a basis for V.

(c) What is the dimension of the subspace V in part (b)?

8. Parts (a), (b) and (c) of this question all refer to the matrix

$$A = \begin{bmatrix} 1 & 2 & -3 & 1 & -3 \\ 2 & 4 & 1 & -3 & 5 \\ 1 & 2 & 2 & 1 & 12 \\ -3 & -6 & -2 & -1 & -20 \end{bmatrix}$$

The reduced echelon form for A is (you do not need to check this)

$$R = \begin{bmatrix} 1 & 2 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find a basis for the column space $C(A)$ of A.

(b) Find a basis for the null space $N(A)$ of A.

(c) If $\mathbf{v} = \begin{bmatrix} 2 \\ -1 \\ -4 \\ 0 \\ 1 \end{bmatrix}$ then $A\mathbf{v} = \begin{bmatrix} 9 \\ 1 \\ 4 \\ -12 \end{bmatrix}$. (You do not need to check this.)

Find all solutions of

$$A\mathbf{x} = \begin{bmatrix} 9 \\ 1 \\ 4 \\ -12 \end{bmatrix}.$$

9. Let

$$\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \mathbf{y} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & -1 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 7 & 0 \\ 4 & 1 \\ 2 & -4 \end{bmatrix}$$

Compute each the following, if possible.

(a) $3\mathbf{x} + 2\mathbf{y}$

(b) $\mathbf{x} \cdot \mathbf{y}$

(c) $A(\mathbf{x} + \mathbf{y})$

(d) $\mathbf{y} + A\mathbf{x}$

(e) $B(A\mathbf{x})$

10. Circle T or F to mark each of the following true or false.

(a) If $\mathbf{v} \cdot \mathbf{w} < 0$ then the angle between $\mathbf{v}$ and $\mathbf{w}$ is less than $90°$. T F

(b) The dot product of two vectors in $\mathbf{R}^3$ is a vector in $\mathbf{R}^3$. T F

(c) Any three vectors in $\mathbf{R}^3$ span $\mathbf{R}^3$. T F

(d) The columns of a matrix are linearly independent if and only if its rank equals the number of columns. T F

(e) For all matrices A, the column space of A equals the column space of the reduced row echelon form of A. T F

(f) For all matrices A, the null space of A equals the null space of the reduced row echelon form of A. T F

(g) If $V = \text{span}(\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k)$, then $\dim(V) \leq k$. T F

(h) If $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are linearly independent vectors in V, then $\dim(V) \geq k$. T F

(i) A system of three equations in four unknowns cannot have a unique solution. T F

(j) A system of four equations in three unknowns cannot have more than one solution. T F