

1. (10 points) In a certain California town, the probability density function for a random day's total rainfall is given by

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ A(x+1)^{-3/2} & \text{if } x \geq 0 \end{cases}$$

where x is measured in millimeters, and A is a positive constant.

- (a) Find A , given that f is a probability density function.

- (b) What is the median amount of daily rainfall in this town?

2. (10 points)

(a) Determine with justification if the series

$$\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$$

converges, and if so, find the sum.

(b) Express $2.0\overline{134} = 2.0134134134134\dots$ as a ratio of integers.

3. (10 points) Determine whether each of the series below converges or diverges. Indicate clearly which tests you use and how you apply them.

(a)
$$\sum_{n=1}^{\infty} \frac{(-4)^{2n}}{n^3 5^n}$$

$$(b) \sum_{n=1}^{\infty} \ln \left(\frac{n}{2n+1} \right)$$

4. (10 points) Determine whether each of the series below converges or diverges. Indicate clearly which tests you use and how you apply them.

(a)
$$\sum_{n=1}^{\infty} \frac{1}{n + \sqrt{n}}$$

$$(b) \sum_{n=1}^{\infty} \sin^2 \frac{\pi}{n}$$

5. (12 points) Suppose that the series $\sum_{n=1}^{\infty} a_n$ converges absolutely.

Decide which of the following series must converge, must diverge, or may either converge or diverge (inconclusive). Circle your answer. You do not need to justify your answers.

(a) $\sum_{n=1}^{\infty} \left(a_n + \frac{1}{n^2} \right)$	Converges	Diverges	Inconclusive
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(b) $\sum_{n=1}^{\infty} (-1)^n a_n$	Converges	Diverges	Inconclusive
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(c) $\sum_{n=1}^{\infty} \frac{1}{1 + a_n^2}$	Converges	Diverges	Inconclusive
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(d) $\sum_{n=1}^{\infty} \frac{ a_n }{n}$	Converges	Diverges	Inconclusive
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(e) $\sum_{n=1}^{\infty} n^2 a_n$	Converges	Diverges	Inconclusive
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(f) $\sum_{n=1}^{\infty} n! a_n$	Converges	Diverges	Inconclusive
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6. (13 points) Find, with complete justification, the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} (3x - 1)^n$$

7. (13 points) In each of the parts below, show all the steps in your reasoning.

- (a) Write $\frac{1}{1+x^2}$ as a power series about 0, and state the interval of convergence. (Hint: use geometric series.)

- (b) Find a power series for $\arctan x$. What is the radius of convergence?

(c) Express the number $\int_0^{0.1} \frac{\arctan x}{x} dx$ as a series.

(d) Find, with complete justification, a partial sum of the series in part (c) that approximates the value of the integral to within 10^{-8} . (You do not need to simplify the sum.)

8. (10 points)

(a) Find, showing all your steps, the Taylor series for e^x with center 0.

(b) Use series to find $\lim_{x \rightarrow 0} \frac{e^{-x^2} - 1}{e^x - x - 1}$. (You may take for granted the fact that the Taylor series for e^x converges to e^x .)