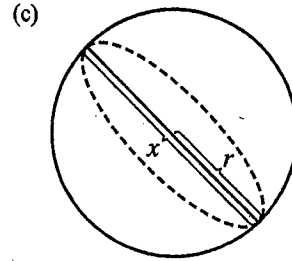


11. (a) Given: the rate of decrease of the surface area is $1 \text{ cm}^2/\text{min}$. If we let t be time (in minutes) and S be the surface area (in cm^2), then we are given that $dS/dt = -1 \text{ cm}^2/\text{s}$.

(b) Unknown: the rate of decrease of the diameter when the diameter is 10 cm. If we let x be the diameter, then we want to find dx/dt when $x = 10 \text{ cm}$.



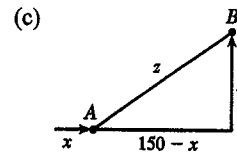
(d) If the radius is r and the diameter $x = 2r$, then $r = \frac{1}{2}x$ and $S = 4\pi r^2 = 4\pi(\frac{1}{2}x)^2 = \pi x^2 \Rightarrow$

$$\frac{dS}{dt} = \frac{dS}{dx} \frac{dx}{dt} = 2\pi x \frac{dx}{dt}$$

(e) $-1 = \frac{dS}{dt} = 2\pi x \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = -\frac{1}{2\pi x}$. When $x = 10$, $\frac{dx}{dt} = -\frac{1}{20\pi}$. So the rate of decrease is $\frac{1}{20\pi} \text{ cm/min}$.

12. (a) Given: at noon, ship A is 150 km west of ship B; ship A is sailing east at 35 km/h, and ship B is sailing north at 25 km/h. If we let t be time (in hours), x be the distance traveled by ship A (in km), and y be the distance traveled by ship B (in km), then we are given that $dx/dt = 35 \text{ km/h}$ and $dy/dt = 25 \text{ km/h}$.

(b) Unknown: the rate at which the distance between the ships is changing at 4:00 P.M. If we let z be the distance between the ships, then we want to find dz/dt when $t = 4 \text{ h}$.



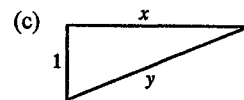
(d) $z^2 = (150 - x)^2 + y^2 \Rightarrow 2z \frac{dz}{dt} = 2(150 - x)\left(-\frac{dx}{dt}\right) + 2y \frac{dy}{dt}$

(e) At 4:00 P.M., $x = 4(35) = 140$ and $y = 4(25) = 100 \Rightarrow z = \sqrt{(150 - 140)^2 + 100^2} = \sqrt{10,100}$. So

$$\frac{dz}{dt} = \frac{1}{z} \left[(x - 150) \frac{dx}{dt} + y \frac{dy}{dt} \right] = \frac{-10(35) + 100(25)}{\sqrt{10,100}} = \frac{215}{\sqrt{101}} \approx 21.4 \text{ km/h}$$

13. (a) Given: a plane flying horizontally at an altitude of 1 mi and a speed of 500 mi/h passes directly over a radar station. If we let t be time (in hours) and x be the horizontal distance traveled by the plane (in mi), then we are given that $dx/dt = 500 \text{ mi/h}$.

(b) Unknown: the rate at which the distance from the plane to the station is increasing when it is 2 mi from the station. If we let y be the distance from the plane to the station, then we want to find dy/dt when $y = 2 \text{ mi}$.

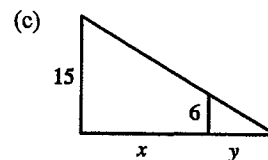


(d) By the Pythagorean Theorem, $y^2 = x^2 + 1 \Rightarrow 2y(dy/dt) = 2x(dx/dt)$.

(e) $\frac{dy}{dt} = \frac{x}{y} \frac{dx}{dt} = \frac{x}{y}(500)$. Since $y^2 = x^2 + 1$, when $y = 2$, $x = \sqrt{3}$, so $\frac{dy}{dt} = \frac{\sqrt{3}}{2}(500) = 250\sqrt{3} \approx 433 \text{ mi/h}$.

14. (a) Given: a man 6 ft tall walks away from a street light mounted on a 15-ft-tall pole at a rate of 5 ft/s. If we let t be time (in s) and x be the distance from the pole to the man (in ft), then we are given that $dx/dt = 5 \text{ ft/s}$.

(b) Unknown: the rate at which the tip of his shadow is moving when he is 40 ft from the pole. If we let y be the distance from the man to the tip of his shadow (in ft), then we want to find $\frac{d}{dt}(x + y)$ when $x = 40 \text{ ft}$.



(d) By similar triangles, $\frac{15}{6} = \frac{x + y}{y} \Rightarrow 15y = 6x + 6y \Rightarrow 9y = 6x \Rightarrow y = \frac{2}{3}x$.

(e) The tip of the shadow moves at a rate of $\frac{d}{dt}(x + y) = \frac{d}{dt}\left(x + \frac{2}{3}x\right) = \frac{5}{3} \frac{dx}{dt} = \frac{5}{3}(5) = \frac{25}{3} \text{ ft/s}$.