QUESTION NOVEMBER 2ND

Question: Rephrased from the student B.W.'s question: "But if the question is asking for a minimum, don't we know already that it has to occur at the only critical number that we found?"

In-class answer: Jose: "Not necessarily, the minimum could also occur on the endpoints. When using the closed interval method, you need to check the images of all critical numbers and endpoints and compare them, the smallest will be the global minimum."

Extended answer: Here is an example where the question asks for a minimum, but the minimum does not occur at a critical number.

Example: The sum of two numbers x, y is 5 and it is known that $x \geq 1$ and $y \geq 1$. What is the minimum value of xy?

Solution: We want to minimize $P = xy$

It is known that $x + y = 5$, so $y = 5 - x$

So $P(x) = x(5 - x) = 5x - x^2$.

Since $x \geq 1$ and $y \geq 1$, the domain of this function is $[1, 4]$. $P(x)$ is continuous on the domain, so we can use the closed interval method.

$P'(x) = 5 - 2x$

$P'(x) = 0$ for $x = \dfrac{5}{2}$

There are no more critical numbers because $P'(x)$ is defined on $[1, 4]$.

$P\left(\dfrac{5}{2}\right) = \dfrac{5}{2} \cdot \dfrac{5}{2} = \dfrac{25}{4}$

$P(1) = 4$

$P(4) = 4$

Since $4 < \frac{25}{4}$, the minimum value of xy is 4.

And as you can see, it is not always the case that the answer to the optimization questions occurs at a critical number. In this case, it occurred at the endpoint (actually at both endpoints in this case).