# Math 220A - Fall 2002
# Homework 3
# Due Friday, Oct. 18, 2002

1. Find the unique weak solution of

$$\begin{cases} u_t + uu_x = 0, \ t \geq 0 \\ u(x,0) = \phi(x) \end{cases}$$

where

$$\phi(x) = \begin{cases} 0 & \text{for } x \leq -1 \\ x+1 & \text{for } -1 \leq x \leq 0 \\ -x+1 & \text{for } 0 \leq x \leq 1 \\ 0 & \text{for } x \geq 1, \end{cases}$$

which satisfies the Rankine-Hugoniot condition and the entropy condition. Show that your solution satisfies the entropy condition. Draw a picture describing your answer, showing the projected characteristics and any shock curves.

2. Find the unique weak solution of

$$\begin{cases} \left(\frac{u^2}{2}\right)_t + \left(\frac{u^3}{3}\right)_x = 0 \\ u(x,0) = \phi(x) \end{cases}$$

where

$$\phi(x) = \begin{cases} 1 & x < 0 \\ 0 & x > 0 \end{cases}$$

3. Find the unique weak solution of

$$\begin{cases} u_t + \left(\frac{u^2}{2}\right)_x = 0 \ , t \geq 0 \\ u(x,0) = \phi(x) \end{cases}$$

which satisfies the Rankine-Hugoniot jump condition and the entropy condition, where the initial data

$$\phi(x) = \begin{cases} 1 & \text{if } x < -1 \\ 0 & \text{if } -1 < x < 0 \\ 3 & \text{if } x > 0. \end{cases}$$

4. Consider the following initial-value problem

$$\begin{cases} u_t - (\cos u)_x = 0 \\ u(x,0) = \phi(x). \end{cases}$$

Find the unique, weak admissible solution which satisfies the Oleinik entropy condition if the initial conditions are given by

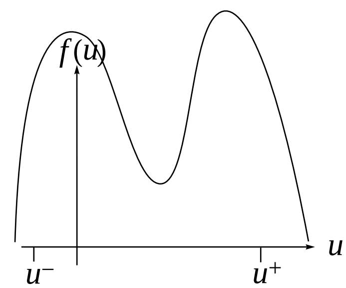
(a)

$$\phi(x) = \begin{cases} \dfrac{\pi}{2} & x < 0 \\ -\dfrac{\pi}{2} & x > 0 \end{cases}$$

(b)

$$\phi(x) = \begin{cases} \pi & x < 0 \\ -\dfrac{\pi}{2} & x > 0 \end{cases}$$

5. Consider $f, u^-, u^+$ shown below.




Consider the initial-value problem

$$\begin{cases} u_t + [f(u)]_x = 0, & t \geq 0 \\ u(x,0) = \phi(x) \end{cases}$$

where

$$\phi(x) = \begin{cases} u^- & x < 0 \\ u^+ & x > 0. \end{cases}$$

Find the weak solution which satisfies the Oleinik entropy condition.