Math 220A
Practice Midterm - Fall 2002

1. Classify the following in terms of degree of nonlinearity:

   (a) $u_t + e^u u_x = x^2$

   (b) $u_t + x^2 u_x = e^u$

   (c) $x^3 u_t + u_x^2 = 1$

   (d) $u_t + x^3 u_x = \sin(x^2)$

   (e) $u_{xt} + \left[\frac{u^2}{2}\right]_x = \cos(u_x)$

2. Find the unique weak solution of

$$\begin{cases} u_t + uu_x = 0 \\ u(x,0) = \phi(x) \end{cases}$$

   where

$$\phi(x) = \begin{cases} 2 & x < 0 \\ 1 & 0 < x < 1 \\ 0 & x > 1 \end{cases}$$

   which satisfies the entropy condition.

3. Solve

$$\begin{cases} u_t + u_x^2 = 0 \\ u(x,0) = -x^2 \end{cases}$$

   Find the time T for which $|u| \to +\infty$ as $t \to T$.

4. Solve

$$\begin{cases} u_x + xu_y - 4u = 0 \\ u(1,y) = y^2. \end{cases}$$

5. Find the unique weak solution of

$$\begin{cases} u_t + u^2 u_x = 0 \\ u(x,0) = \phi(x) \end{cases}$$

   where

$$\phi(x) = \begin{cases} 0 & x < 0 \\ 1 & 0 < x < 2 \\ 0 & x > 2 \end{cases}$$

   which satisfies the Oleinik entropy condition.

6. Let f, g be C^∞ functions. Consider the initial value problem

$$\begin{cases} [g(u)]_t + [f(u)]_x = 0 \\ u(x,0) = \phi(x) \end{cases}$$

(a) Give a definition for a strong solution of this initial value problem.

(b) Give a definition for a weak solution of this initial value problem.

(c) Prove that any strong solution is in fact a weak solution.

7. (a) Find the general solution of

$$u_{tt} + 2u_{xt} - 3u_{xx} = 0.$$

(b) Find the solution of the initial-value problem,

$$\begin{cases} u_{tt} + 2u_{xt} - 3u_{xx} = 0 \\ u(x,0) = \phi(x) \\ u_t(x,0) = \psi(x). \end{cases}$$