MATH 20 INTEGRAL REVIEW

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0.1. **Main idea.** FTC implies: for every rule of differentiation, there is a corresponding rule of anti-differentiation, and therefore a corresponding rule of integration.

0.2. **Basic Forms.** Don't forget $\int \frac{dx}{1+x^2}, \int \sec^2(x)dx, \ldots$

0.3. ***u*-substitution.** From the chain rule. Some tricks: look for du; long division; complete the square $ax^2+bx+c = a((x+\frac{b}{2a})^2 + \frac{c}{a} - \frac{b^2}{4a^2})$; to get rid of $\sqrt[n]{ax+b}$, do the substitution $u = \sqrt[n]{ax+b}$, solve for $x = \frac{u^n - b}{a}$, so $dx = \frac{n}{a}u^{n-1}du$.

0.4. **Powers of trig: $\int \sin^n(x)\cos^m(x)dx$.** The idea: turn everything into a polynomial in u = trig function times du. So odd power of sin, pull off one sin, turn everything into cos; same for odd power of cos; use $\sin^2(x)+\cos^2(x) = 1$ to convert. (Divide by $\cos^2(x)$ to get the sec and tan version, or by $\sin^2(x)$ to get the csc, cot.) If they're both even powers, use $\sin^2(x) = \frac{1-\cos(2x)}{2}$, $\cos^2(x) = \frac{1+\cos(2x)}{2}$, or reduction formulas

$$\int \sin^n(x) = -\frac{1}{n}\cos(x)\sin^{n-1}(x) + \frac{n-1}{n}\int \sin^{n-2}(x)dx$$

which you derive from parts, $u = \sin^{n-1}(x), dv = \sin(x)dx$; likewise for cos. For powers of sec, tan, try to pull off $\sec^2(x)$ and convert to $\tan(x)$, or pull off $\sec(x)\tan(x)$ and convert to $\sec(x)$. If the powers don't work out right, you can try to convert to sec and use

$$\int \sec^n(x) = \frac{1}{n-1}\sec^{n-2}(x)\tan(x) + \frac{n-2}{n-1}\int \sec^{n-2}(x)dx$$

This is again from parts. There's also a tan formula (derived without parts!) Finally, these are difficult (but $\int \tan, \int \cot$ are not):

$$\int \sec(x)dx = \ln|\sec(x)+\tan(x)| + C, \int \csc(x)dx = \ln|\csc(x)-\cot(x)| + C.$$

0.5. **(Inverse) Trig substitution.** Remember $4 - x^2 = (\sqrt{4-x^2})^2$. $\sqrt{a^2-x^2} \Rightarrow x = a\sin\theta$; $\sqrt{x^2-a^2} \Rightarrow x = a\sec\theta$; $\sqrt{x^2+a^2} \Rightarrow x = a\tan\theta$. Draw the triangle to get the radical; back substitute using the triangle or solving for θ; remember $\sin 2\theta = 2\sin\theta\cos\theta$.

0.6. **Partial fractions.** 1. Long division; 2. Factor denominator; 3. Write out terms

$$\frac{2x+1}{(x-1)^3(x^2+1)^2} = \frac{A}{(x-1)^3} + \frac{B}{(x-1)^2} + \frac{C}{x-1} + \frac{Dx+E}{(x^2+1)^2} + \frac{Fx+G}{x^2+1}.$$

Remember $x^2 = x \times x$ is a repeated linear factor, not a quadratic. 4. Solve for $A, B, \ldots$ by equating coefficients or plugging in points. 5. Split and integrate.

0.7. **Integration by Parts.**

$$\int udv = uv - \int vdu.$$

Choosing the right parts is hard – choose u so du is simpler, dv easy to integrate. If it doesn't work try to borrow from the obvious u to dv, and vice versa.

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