

Math 20: Midterm 2

Tuesday, 02/22/2011

- Complete the following problems. You may use any result from class you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- This is a closed-book exam. No calculators or other electronic aids will be permitted. You may have one sheet of handwritten notes, with writing on both sides. Page dimensions must not exceed 8.5 by 11 inches.
- In order to receive full credit, *you must show all of your work and justify your answers.* Your answer should be clearly labeled.
- If you need extra room, use the back sides of each page. Staple any scratch paper to your exam.
- Here are some useful formulas:
 - $\sin(2x) = 2 \sin(x) \cos(x)$
 - $\cos(2x) = \cos^2(x) - \sin^2(x) = 2 \cos^2(x) - 1 = 1 - 2 \sin^2(x)$
- Please sign the following:

“On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination.

Name: _____

Signature: _____

(1) _____ (/20 points)

(2) _____ (/10 points)

(3) _____ (/10 points)

(4) _____ (/10 points)

(5) _____ (/15 points)

(6) _____ (/15 points)

(7) _____ (/10 points)

Total. _____ (/90 points)

- (1) (20 points) Consider the list of integrals below. Choose any 4 and integrate. Remember to interpret improper integrals as necessary. Do NOT resort to approximate integration. Do NOT just cite a formula you have memorized, give some work or an appropriate argument for each of these problems. Be sure to indicate which 4 you want graded, otherwise the grader will pick the first 4 integrals they see.

$$\begin{array}{ll} \int x \cos(x) dx & \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin(x^3) + 5 dx \\ \int_0^1 \frac{dx}{x\sqrt{x}} & \int \frac{dx}{x^2 + 4} \\ \int \frac{1}{x^2\sqrt{4-x^2}} dx & \int \frac{x}{x+1} dx \\ \int_0^1 \sin^{-1}(x) dx & \int \sin^2(x) \cos^3(x) dx \end{array}$$

(this page intentionally blank as overflow for problem 2)

(2) (10 points) This question is about partial fractions.

(a) (2 points) Write out the form of the partial fractions decomposition of

$$\frac{x^2 + 3}{x^4 - x^2}$$

DO NOT DETERMINE COEFFICIENTS!

(b) (2 points) Write out the form of the partial fractions decomposition of

$$\frac{1}{x^4 + x^2}$$

DO NOT DETERMINE COEFFICIENTS!

(c) (2 points) Write out the form of the partial fractions decomposition of

$$\frac{x + 2}{x^3 - x}$$

DO NOT DETERMINE COEFFICIENTS!

(d) (4 points) Integrate

$$\int \frac{2}{t^2 - t} dt.$$

- (3) (10 points) This question is about finding the area between two curves.
- (a) Sketch the region bounded by the curves $y = x^2 - 2x$ and $y = x + 4$
 - (b) Compute the area of the region from (a).

- (4) (10 points) This question is about approximate integration for the integral

$$\int_1^3 \frac{1}{x} dx.$$

- (a) (5 points) Use the trapezoid rule with $n = 6$ to approximate the value of the integral. You do not need to simplify your answer, just write out the expression that approximates the value of the integral right before the point where you'd reach for your calculator.

- (b) (5 points) Estimate the maximum error in this approximation.

(5) (15 points) This question is about improper integrals.

(a) (5 points) Compute

$$\int x e^{-x} dx.$$

(b) (5 points) Compute

$$\int_1^{\infty} x e^{-x} dx.$$

(c) (5 points) Does the integral

$$\int_1^{\infty} \frac{x}{e^x + x + 1} dx$$

converge or diverge? Explain carefully.

(6) (15 points) Compute

$$\int \frac{x^2}{\sqrt{9-x^2}} dx$$

(7) (10 points) True/False. No justification needed

(a) (2 points) True or False: If f is continuous on $[a, b]$ then $\int_a^b xf(x) dx = x \int_a^b f(x) dx$.

(b) (2 points) True or False: All continuous functions have derivatives.

(c) (2 points) True or False: All continuous functions have antiderivatives.

(d) (2 points) True or False: If $0 \leq g(x) \leq f(x)$ on $[1, \infty)$ and $\int_1^\infty f(x) dx$ diverges, then $\int_1^\infty g(x) dx$ diverges too.

(e) (2 points) True or False: $\frac{d}{dx} \int_a^b f(x) dx = f(x)$.