

Math 20: Midterm 1

Tuesday, 02/01/2011

- Complete the following problems. You may use any result from class you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- This is a closed-book exam, however you may have one sheet of notes on 8.5 by 11 inch paper with handwriting on both sides. No calculators or other electronic aids will be permitted.
- In order to receive full credit, *you must show all of your work and justify your answers.* Your answer should be clearly labeled.
- If you need extra room, use the back sides of each page. Staple any scratch paper to your exam.
- The following formulas may be of use on this exam:

$$\sin(2x) = 2 \sin x \cos x.$$

$$\cos(2x) = \cos^2(x) - \sin^2(x).$$

$$\text{Volume of a cylinder : } V = \pi r^2 h.$$

$$\text{Volume of a cone : } V = \frac{\pi}{3} r^2 h.$$

$$\text{Circumference of a circle : } C = \pi d = 2\pi r.$$

- Please sign the following:
“On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination.

Full Name: _____

Circle your lecture time: 9:30 / 11:00

Signature: _____

(1) _____ (/10 points)

(2) _____ (/10 points)

(3) _____ (/10 points)

(4) _____ (/25 points)

(5) _____ (/10 points)

(6) _____ (/25 points)

(7) _____ (/10 points)

Total. _____ (/100 points)

- (2) (10 points) A cylindrical can without a top is made to contain 64 cm^3 of liquid. Find the dimensions that will minimize the surface area to make the can. Justify that your dimensions yield a minimum.

(3) (10 points) Consider the following three integrals:

(a) $\int_0^1 e^{\sqrt{x}} dx$.

(b) $\int_0^1 2xe^x dx$.

(c) $\int_0^{\frac{\pi}{2}} \sin(2x)e^{\sin(x)} dx$.

Which of the above integrals are equal? (*hint: you do not need to completely evaluate these integrals to real numbers to decide this!*) .

- (4) (25 points) This question is about the Fundamental theorem of Calculus
- (a) (5 points) State both parts of the Fundamental Theorem of Calculus as accurately as possible.

- (b) (5 points) Suppose $F(x) = \int_0^x e^t - 1 dt$. Compute $F(0)$, $F(1)$ and $F(2)$ explicitly.

(c) (5 points) Compute $F'(x)$.

(d) (5 points) On what subinterval of $[0, \infty)$ is $F(x)$ increasing?

(e) (5 points) Suppose $G(x) = \int_0^{\frac{1}{x}} e^t - 1 dt$. Compute $G'(x)$.

- (5) (10 points) This question is about the definition of the integral.
- (a) (5 points) The following limit may be regarded as a definite integral. Determine a region whose area is equal to this limit, and write this limit as a definite integral.

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\pi}{4n} \tan\left(\frac{i\pi}{4n}\right).$$

- (b) (5 points) Evaluate the limit in part (a) by evaluating the integral you found. (if you are unsure of your answer for (a), then compute $\int \tan(x)dx$ for a one point deduction.)

- (6) (25 points) Evaluate the integral, using any technique. For indefinite integrals, report the most general antiderivative.

(a) (5 points)

$$\int 3x^2 + 1 + \frac{1}{x} + \frac{5}{x^2} dx.$$

(b) (5 points)

$$\int_1^2 \frac{v^3 + 3v^6}{v^4} dv.$$

(c) (5 points)

$$\int t^2 \sqrt{2+t} dt.$$

(d) (5 points)

$$\int \frac{x}{\sqrt{a^2 - x^2}} dx, \text{ where } a \text{ is a constant.}$$

(e) (5 points)

$$\int_{-5}^5 x^3 + \sqrt{25 - x^2} dx.$$

(7) (10 points) True/False and short answer. No justification is needed. Pictures may help you decide.

(a) (2 points) True or false:

$$\int_0^1 \frac{1}{x+1} dx = \ln|x+1| \Big|_0^1 = \ln|1+1| - \ln|0+1| = \ln 2.$$

(b) (2 points) True or false: If $f(x) < 0$ on $[0, 1]$, then $\int_0^1 f(x) dx < 0$.

(c) (2 points) True or false: If $\int_0^4 f(x) dx = 10$, then $\int_0^2 f(2x) dx = 20$.

(d) (2 points) True or false: $\frac{1}{1+a}x^{1+a}$ is an antiderivative of x^a for all a .

(e) (2 points) Write $4 + 6 + 8 + 10 + 12$ in sigma notation.