Math 151 Homework 2

Due: Friday, January 24, 2003

Problem 1

A family has three children, each independently equally likely to be a boy or girl.

(a) What is the probability that they have two girls, given that the youngest is a boy?

(b) What is the probability that they have two girls, given that they have at least one boy?

(c) What is the probability that they have a girl, given that they have at least two boys?

Listing all possible outcomes may be helpful.

Problem 2

The Angels play the Giants in the World Series. A team wins the Series by being the first to win four games; at that point the Series ends. Suppose that the Angels win each game with probability p, independently of all other games.

(a) For n=4, 5, 6, 7, what is the probability that the Angels win the Series in *exactly* n games? Hint: consider what must happen in game n, and what must happen in the first $n-1$ games.

(b) Given that the Angels win the Series, what is the probability that the Series lasts exactly 5 games?

Problem 3

Dr. Evil has created n Fembots, whose birthdays are all different. Consider also a group of n humans, each of whom has a birthday equally likely to be any of the D days in a year, independently of anyone else's birthday.

(a) What is the probability that there exist a human and a Fembot who share the same birthday?

(b) Find a lower bound on that probability, using the fact that $1 + x \leqslant e^x$ for all $x \in \mathbb{R}$.

(c) For what values of n does that lower bound exceed 1/2? (State your answer in terms of D)

To clarify, D is the full number of days in a year (365), but please give your answers in terms of D (not 365), so we see how hard/easy it is to generate collisions in more general situations (e.g. what if you had $D = 1000000$ buckets).

This is actually a rough analysis of the "birthday attack" against digital signature schemes.

Problem 4

Archie, Beth, and Chuck shoot at a target. They hit it with probabilities 1/2, 2/3, and 3/4, respectively. Suppose that they each independently fire one shot.

(a) Find the probability that exactly one shot hits the target.

(b) Given that exactly one shot hits, what is the probability that it's Archie's shot?

(c) Are the events {Archie hits} and {Exactly one shot hits} independent?