

Math 151 Homework 1

Due: Friday, January 17, 2003

Problem 1

(Durrett 1.5.11) Pick a number at random between 0 and 999, with all numbers equally likely. Here we regard, say, 7 as 007. Find the probability that

- (a) all three digits are different
- (b) exactly two digits are the same
- (c) all three are the same.

You may want to use a fact that relates the three parts.

Problem 2

Playing Lotto, you have chosen a set of six numbers from among $\{1, 2, \dots, 51\}$. From a bin of balls numbered 1 through 51, the lottery machine will draw, without replacement, 6 *winning numbers* and then 1 *bonus number*. All possible draws are equally likely.

- (a) How many different draws can the machine make?
- (b) How many of those draws have exactly 4 of your numbers as winning numbers, and none of your numbers as the bonus number? Find the probability of this “4 correct, no bonus” event.
(Answer: ≈ 0.000788)
- (c) How many of those draws have exactly 3 of your numbers as winning numbers, and 1 of your numbers as the bonus number? Find the probability of this “3 correct, plus bonus” event.
(Answer: ≈ 0.001051)

To reiterate, you have chosen your numbers, and they are fixed. What we are treating as uncertain is the machine’s draw. In each part, we are counting machine draws, not human plays.

For a related problem see Durrett’s Example 4.2 on page 24.

Problem 3

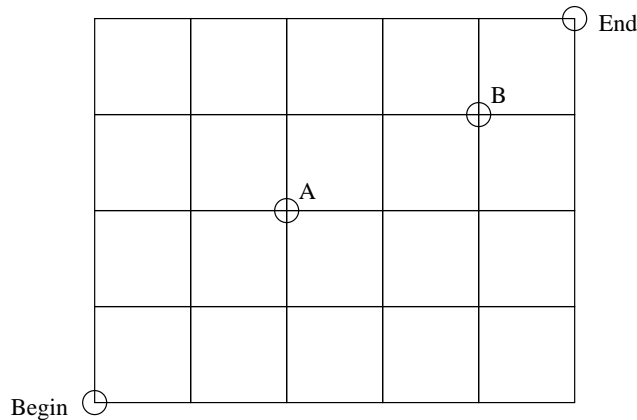
Events A , B , and C are defined in a sample space. Express the following probabilities in terms of $P(A)$, $P(B)$, $P(C)$, $P(A \cap B)$, $P(A \cap C)$, $P(B \cap C)$, and $P(A \cap B \cap C)$.

- (a) The probability that exactly two of A, B, C occur.
- (b) The probability that exactly one of these events occurs.
- (c) The probability that none of these events occur.

Algebraic / tabular / Venn-diagram derivations are all acceptable, but each final answer should be a formula.

Problem 4

A bug moves along this grid, from the node marked Begin to the node marked End.



Each move it makes is either one step upward or one step to the right.

- (a) How many different paths from Begin to End are possible?
Hint: Each such path consists of 9 steps, of which 4 are upward.

The bug is equally likely to take any of the paths from Begin to End. Find the probability that:

- (b) It goes through node A.
- (c) It goes through node B.
- (d) It goes through A and B.
- (e) It goes through A or B.
- (f) It goes through B but not A.