Assignment 3: Assigned Wed 04/18. Due Wed 04/25

1. **Section 2.4:** 50, 51, 52
2. (a) Let $T : \mathbb{R}^m \to \mathbb{R}^n$ be a linear transformation. If $T(\vec{x}) = T(\vec{y})$ show that $\vec{x} - \vec{y} \in \ker(T)$. Conclude that T is injective if and only if $\ker(T) = \{\vec{0}\}$.
   (b) If $T : \mathbb{R}^m \to \mathbb{R}^n$ is a linear transformation with rank m, show that T is injective.
3. (a) Let R be a row operation and T and $m \times n$ matrix. Show that the kernel of T and $R(T)$ are the same.
   (b) Given an example to show that the image of T and $R(T)$ are *not* the same.
   (c) Let C be a column operation and T an $m \times n$ matrix. Show that the *image* of T and $C(T)$ are the same.
   (d) Give an example to show that the kernel of T and $C(T)$ are *not* the same.
4. **Section 3.1:** 16, 38
5. **Section 3.2:** 36, 38, 50

Assignment 4: Assigned Wed 04/25. Due Wed 05/02

1. **Section 3.4:** 18, 30, 38, 74, 76

Assignment 5: Assigned Fri 05/04. Due Wed 05/09

1. **Section 4.1** 14, 30, 40, 58
2. **Section 4.2** 4, 16, 70

Assignment 6: Assigned Wed 05/09. Due Wed 05/16

1. If $V \subseteq \mathbb{R}^n$ is a subspace, show that $(V^\perp)^\perp = V$.
2. Let $V \subseteq \mathbb{R}^n$ be a subspace. If $\vec{x} \in \mathbb{R}^n$, show that $\|P_V(\vec{x})\| \leqslant \|\vec{x}\|$. What does this mean geometrically?
3. **Section 5.1** 10, 12, 14, 26, 36
4. **Section 5.2** 6, 30, 34

Assignment 7: Assigned Wed 05/16. Due Wed 05/23

1. Show that $(\ker A)^\perp = \operatorname{im}(A^t)$.
2. Let $d, a_1, \dots, a_n \in \mathbb{R}$, and P be the 'hyperplane' defined by $P = \{\vec{x} \in \mathbb{R}^n \mid a_1x_1 + \cdots + a_nx_n = d\}$. Show that the shortest distance between P and the origin is $\frac{d}{\sqrt{a_1^2 + \cdots a_n^2}}$. [Hint: First choose $n = 2$, and draw a picture. Then try and generalize your argument.]
3. (a) Let $B = \{\vec{v}_1, \dots, \vec{v}_n\}$ be an orthonormal basis of $\mathbb{R}^n$. Let $\vec{x}, \vec{y} \in \mathbb{R}^n$, and $\alpha_1, \dots, \alpha_n$, $\beta_1, \dots, \beta_n$ be such that $\vec{x} = \sum_i \alpha_i \vec{v}_i$ and $\vec{y} = \sum_i \beta_i \vec{v}_i$. Show that $\vec{x} \cdot \vec{y} = \sum_i \alpha_i \beta_i$. [Notice that $(\vec{x})_B = \vec{\alpha}$, and $(\vec{y})_B = \vec{\beta}$. This problem says that the dot product $\vec{x} \cdot \vec{y}$, where $\vec{x}$ and $\vec{y}$ are in standard coordinates is the same as $(\vec{x})_B \cdot (\vec{y})_B$, where coordinates are in basis B.]
   (b) Give an example to show that $\vec{x} \cdot \vec{y} \neq (\vec{x})_B \cdot (\vec{y})_B$ if B is not orthonormal.
4. (a) Let A be an $m \times n$ matrix, and $\vec{b} \in \mathbb{R}^m$. Let $V = \operatorname{im} A$ (note that V is a subspace of $\mathbb{R}^m$), and suppose $\ker A = \{\vec{0}\}$. Show that $P_V(\vec{b}) = A(A^t A)^{-1} A^t \vec{b}$. [Hint: Least squares ...]
   (b) Let $\{\vec{v}_1, \dots, \vec{v}_n\}$ be a basis of some subspace $V \subseteq \mathbb{R}^m$, and A be the $m \times n$ matrix with the vectors $\vec{v}_i$'s as columns. Find a matrix M (in terms of the matrix A) such that $P_V(\vec{b}) = Mb$. [This tells you how to find the matrix of an orthogonal projection when you have a basis which is *not* orthogonal. In reality however, it is better to use Gram-Schmidt to convert your basis to an orthonormal basis, and then use the formula from class]
5. **Section 5.3:** 4, 32, 50
6. **Section 5.4:** 8, 10, 20