

Math 103 Midterm

May 2nd, 2007

Time: 50 mins
Total: 50 points

This is a closed book test. Calculators and other computational aids are strictly forbidden. Lucky charms won't help you, but feel free to use them. Cell phones won't help you either, but do not feel free to use them. Good luck ☺

1. Let $\vec{v}_1 = \vec{e}_1 + \vec{e}_2 + \vec{e}_3$, $\vec{v}_2 = \vec{e}_1 - \vec{e}_2 + \vec{e}_3$, and $\vec{v}_3 = \vec{e}_1 + \vec{e}_2 - \vec{e}_3$ be three vectors in \mathbb{R}^3 .

5

(a) Let A be the 3×3 matrix with \vec{v}_1 , \vec{v}_2 and \vec{v}_3 as columns. Compute the inverse of A .

5

(b) Show that $B = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is a basis of \mathbb{R}^3 .

5

(c) Suppose $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is a linear transformation such that $T(\vec{v}_1) = \vec{v}_1 + \vec{v}_2 + \vec{v}_3$, $T(\vec{v}_2) = \vec{v}_1 - \vec{v}_2 + \vec{v}_3$, and $T(\vec{v}_3) = \vec{v}_1 + \vec{v}_2 - \vec{v}_3$. Find the matrix of T in coordinates with respect to the basis B .

5

(d) If T is the transformation from the previous subpart, find the matrix of T in standard coordinates. [You can leave your answer as a product of matrices.]

10

2. Let U and V be two k dimensional subspaces of \mathbb{R}^n . Suppose $U \subseteq V$. Show that $U = V$.

10

3. Find 3 *different* examples of a *non-diagonal* 3×3 matrix A such that $A^2 = I$. [Recall a *diagonal* matrix is a matrix whose only non-zero entries are on the diagonal that starts from the top left, and goes to the bottom right. A *non-diagonal* matrix is not a diagonal matrix. For example, the identity matrix is a diagonal matrix.]

HINT: Note that $A^2 = I$ is the same as saying $A = A^{-1}$. Now think about how you compute inverses of a matrix using row operations.

5

4. (a) Does there exist a 4×4 matrix A such that $\text{im}(A) = \ker(A)$? If yes, give an example. If no prove that no such matrix can exist.

5

(b) Does there exist a 5×5 matrix A such that $\text{im}(A) = \ker(A)$? If yes, give an example. If no prove that no such matrix can exist.