Lambda calculi for semantic theories

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1 Fragment

Definition 1 (Semantic types)**.** The smallest set *Types* such that

i. $e \in \mathit{Types}$

ii. $t \in \mathit{Types}$

iii. If $\sigma, \tau \in \mathit{Types}$, then $\langle \sigma, \tau \rangle \in \mathit{Types}$

Definition 2 (Well-formed expressions)**.** The smallest set *WFF* such that ('$\alpha : \sigma$' as '$\alpha \in \mathit{WFF}$, has type σ'):

i. Every constant is in *WFF*:

  a. $\langle e, t \rangle : \{\mathbf{run}, \mathbf{walk}, \mathbf{student}, \mathbf{unicorn}\}$
  b. $e : \{\mathbf{sandy}, \mathbf{kim}, \mathbf{jesse}\}$
  c. $\langle e, \langle e, t \rangle\rangle : \{\mathbf{find}, \mathbf{lose}, \mathbf{eat}, \mathbf{love}, \mathbf{be}\}$
  d. $\langle t, \langle e, t \rangle\rangle : \{\mathbf{believe}, \mathbf{deny}\}$
  e. $\langle\langle e, t \rangle, \langle e, t \rangle\rangle : \{\mathbf{rapidly}, \mathbf{allegedly}, \mathbf{fail\_to}\}$
  f. $\langle t, t \rangle : \{\mathbf{necessarily}\}$
  g. $\langle\langle e, t \rangle, \langle\langle e, t \rangle, t \rangle\rangle : \{\mathbf{the}, \mathbf{every}, \mathbf{a}, \mathbf{most}, \mathbf{no}\}$

ii. For every type τ, we have an infinite stock of variables of type τ. Variables are in *WFF*.

iii. If $\alpha : \langle \sigma, \tau \rangle$ and $\beta : \sigma$, then $(\alpha\ \beta) : \tau$

iv. If $\alpha : \tau$ and χ is an variable of type σ, then $(\lambda \chi\ \alpha) : \langle \sigma, \tau \rangle$

Definition 3 (Denotation domains)**.** Each semantic type has a corresponding denotation domain:

i. The domain of type e is D_e, the set of entities.

ii. The domain of type t is $D_t = \{\mathtt{T}, \mathtt{F}\}$

iii. The domain of a functional type $\langle \sigma, \tau \rangle$ is the set of all functions from D_σ into D_τ.

Definition 4 (Models)**.** A model is a pair $\mathscr{M} = \langle \mathbf{D}, \|\cdot\|^{\mathbf{M}} \rangle$, where $\mathbf{D}$ is the infinite hierarchy of domains defined in def. 3, and $\|\cdot\|^{\mathbf{M}}$ is a valuation function interpreting the constants of the language, constrained so that $\|\alpha\|^{\mathbf{M}} \in D_\sigma$ iff α is of type σ.

Definition 5 (Assignment functions)**.** Given a model $\mathscr{M} = \langle \mathbf{D}, \|\cdot\|^{\mathbf{M}} \rangle$ and well-formed expressions *WFF*, a function g is an assignment function iff g is a total mapping from the set of variables in *WFF* into $\mathbf{D}$ where $g(\chi) \in D_\sigma$ iff χ is of type σ.

Definition 6 (Interpretation)**.** The interpretation function given model $\mathscr{M} = \langle \mathbf{D}, \|\cdot\|^{\mathbf{M}} \rangle$ and assignment function g is $[\![\cdot]\!]^{\mathbf{M},g} : \mathit{WFF} \mapsto \mathbf{D}$.

- $[\![\varphi]\!]^{\mathbf{M},g} = g(\varphi)$ if φ is a variable
- $[\![\varphi]\!]^{\mathbf{M},g} = \|\varphi\|^{\mathbf{M}}$ if φ is a constant
- $[\![(\varphi\ \psi)]\!]^{\mathbf{M},g} = [\![\varphi]\!]^{\mathbf{M},g}([\![\psi]\!]^{\mathbf{M},g})$
- For $(\lambda \chi\ \varphi) : \langle \sigma, \tau \rangle$, $[\![(\lambda \chi\ \varphi)]\!]^{\mathbf{M},g} =$ the f from D_σ into D_τ such that for all ☺ $\in D_\sigma$, $f(☺) = [\![\varphi]\!]^{\mathbf{M}g[\chi \mapsto ☺]}$

2 Is it a type

For each of the following, say whether it is a type according to def. 1:

(1) e

(2) t

(3) σ

(4) $\langle e\rangle$

(5) $\langle\langle e,t\rangle, e\rangle$

(6) $\langle t,\langle t,\langle t,t\rangle\rangle\rangle$

(7) $\langle e,e,t\rangle$

(8) $e,\langle e,t\rangle$

(9) $\langle t,e\rangle$

3 What kind of expression is it

For each of the following, say whether it is a constant, a variable, a complex expression, or ill-formed. If it is well-formed, give its type.

(10) **run**

(11) **jog**

(12) **necessarily**

(13) x

(14) φ

(15) $(\varphi\ \psi)$

(16) (**walk kim**)

(17) $(\lambda x\ ((\textbf{find}\ x)\ x))$

(18) $(\lambda x\ (\textbf{run}\ x))$

(19) $(\lambda x\ (\textbf{run}\ y))$

(20) $(\lambda x\ \textbf{run})$

(21) $((\lambda x\ (\textbf{walks}\ x))\ \textbf{kim})$

(22) ((**walk kim**) **sandy**)

4 Denotations for (some of) the constants

(23) $||\textbf{sandy}||^{\mathbf{M}} =$ 😫

(24) $||\textbf{kim}||^{\mathbf{M}} =$ 🙁

(25) $||\textbf{jesse}||^{\mathbf{M}} =$ 🤔

(26) $||\textbf{student}||^{\mathbf{M}} =$ the function $f \in D_{\langle e,t \rangle}$ such that, for all ☺ $\in D_e$, $f($☺$) = \mathrm{T}$ if ☺ $\in \{$🙁, 🤔$\}$, else F

(27) $||\textbf{fail\_to}||^{\mathbf{M}} =$

(28) $||\textbf{rapidly}||^{\mathbf{M}} =$

(29) $||\textbf{necessarily}||^{\mathbf{M}} =$

(30) $||\textbf{every}||^{\mathbf{M}} =$

(31) $||\textbf{a}||^{\mathbf{M}} =$

(32) $||\textbf{most}||^{\mathbf{M}} =$

(33) $||\textbf{no}||^{\mathbf{M}} =$

(34) $||\textbf{the}||^{\mathbf{M}} =$

5 Interpretation

Use def. 6 to calculate the meanings of the following relative to the $\|\cdot\|^{\mathbf{M}}$ defined in sec. 4 and the assignment function

$$g = \begin{bmatrix} x & \mapsto & \text{😫} \\ y & \mapsto & \text{🙁} \\ z & \mapsto & \text{🤔} \end{bmatrix}$$

(35) $[\![\textbf{sandy}]\!]^{\mathbf{M},g}$

(36) $[\![x]\!]^{\mathbf{M},g}$

(37) $[\![(\textbf{student sandy})]\!]^{\mathbf{M},g}$

(38) $[\![(\textbf{student } x)]\!]^{\mathbf{M},g}$

(39) $[\![(\textbf{student } z)]\!]^{\mathbf{M},g}$

(40) $[\![(\textbf{student } z)]\!]^{\mathbf{M} g[z \mapsto \text{😫}]}$

(41) $[\![(\lambda x\ (\textbf{student } x))]\!]^{\mathbf{M},g}$

(42) $[\![(\lambda y\ (\textbf{student } x))]\!]^{\mathbf{M},g}$

(43) ((**fail_to run**) **sandy**)

6 Axioms

The expression $\varphi[x \triangleright a]$ says 'φ with all occurrences of x turned into a.' I assume that $\varphi[x \triangleright a]$ has no effect if x is not a variable, and that $\varphi[x \triangleright a]$ is permitted iff it does not result in accidental binding, that is, a term in which there is a λ that binds more variables than it did prior to substitution. (For the full definition: Carpenter 1997, *Type-Logical Semantics*, p. 44.)

Definition 7 (α conversion). $(\lambda x\ \varphi) \overset{\alpha}{\Longrightarrow} (\lambda y\ \varphi[x \triangleright y])$ (permitted iff $\varphi[x \triangleright y]$) is permitted)

Definition 8 (β conversion). $((\lambda \chi\ \varphi)\ \psi) \overset{\beta}{\Longrightarrow} \varphi[\chi \triangleright \psi]$ (permitted iff $\varphi[\chi \triangleright \psi]$ is permitted)

Definition 9 (η conversion). $(\lambda \chi\ (\varphi \chi)) \overset{\eta}{\Longrightarrow} \varphi$ (permitted iff χ is not free in φ)

The axioms all preserve meaning in the sense of the models described above. The special provisions are designed to ward off meaning changes.

(44) α conversion of x to y?

a. $\lambda x\ (\mathbf{see}\ x)$

b. $\lambda x\ ((\mathbf{see}\ x)\ y)$

(45) β conversion?

a. $((\lambda x\ (\mathbf{happy}\ x))\ \mathbf{kim})$

b. $((\lambda x\ (\lambda y\ ((\mathbf{see}\ x)\ y)))\ (\mathbf{friend\text{-}of}\ y))$

(46) η conversion?

a. $(\lambda x\ (\mathbf{dog}\ x))$

b. $(\lambda x\ ((\mathbf{see}\ x)\ x))$

7 Reasoning

Definition 10 (Generalized entailment for meanings)**.** For all domains $\mathbf{D}$ and meanings $a, b \in \mathbf{D}$:

i. If $a, b \in D_e$, then $a \sqsubseteq b$ iff $a = b$
ii. If $a, b \in D_s$, then $a \sqsubseteq b$ iff $a = b$
iii. If $a, b \in D_t$, then $a \sqsubseteq b$ iff $a = \mathtt{F}$ or $b = \mathtt{T}$
iv. If $a, b \in D_{\langle\sigma,\tau\rangle}$, $a \sqsubseteq b$ iff for all $d \in D_\sigma$, $a(d) \sqsubseteq b(d)$

Definition 11 (Generalized entailment for forms)**.** For all models $\mathbf{M}$, assignments g, and expressions α and β, $\alpha \Rightarrow \beta$ iff $[\![\alpha]\!]^{\mathbf{M},g} \sqsubseteq [\![\beta]\!]^{\mathbf{M},g}$.

Definition 12 (Non-entailment, $\not\Rightarrow$)**.** If it's false that $\alpha \Rightarrow \beta$, then $\alpha \not\Rightarrow \beta$

Definition 13 (Synonymy, $\equiv$)**.** If $\alpha \Rightarrow \beta$ and $\beta \Rightarrow \alpha$, then $\alpha \equiv \beta$.

Definition 14 (Contradiction, $|$)**.** $\alpha \mid \beta$ iff $\alpha \Rightarrow \neg\beta$

Definition 15 (Antonymy)**.** [not sure ... very tricky]

8 Meaning postulates

In theories of the sort described here, lexical semantics is done via meaning postulates, which seek to ensure the needed entailments of lexical items and capture relationships between them:

(47) For all models $\mathbf{M} = \langle \mathbf{D}, ||\cdot||^{\mathbf{M}} \rangle$ appropriate for English:

a. $||\mathbf{dog}||^{\mathbf{M}} \sqsubseteq ||\mathbf{mammal}||^{\mathbf{M}}$
b. $||\mathbf{dog}||^{\mathbf{M}} \sqsubseteq ||\neg\mathbf{cat}||^{\mathbf{M}}$
c. $||\mathbf{couch}||^{\mathbf{M}} = ||\mathbf{sofa}||^{\mathbf{M}}$
d. For all $P \in D_{\langle e,\langle s,t\rangle\rangle}$ and $a \in D_e$, $||\mathbf{find}||^{\mathbf{M}}(P)(a) \sqsubseteq ||\mathbf{exist}||^{\mathbf{M}}(P)$ (cf. **seek**)

9 Some consequences

(48) It should bug you to see $(\lambda x\ (\textbf{redundant}\ x))$. Why?

(49) Representational details in the lambda calculus matter only if they impact denotations. Why?

(50) Meaning and reference: all meaning is reference in the extended sense of identifying objects in the domains.

(51) Confluence: the order of β-reductions does not matter. All licit reduction paths lead to the same formula and hence to the same meaning. Why?

(52) Too few meanings! We need to have a richer space that $\{\mathtt{T}, \mathtt{F}\}$. For the $\langle t, t\rangle$ expressions, there are only 4 functions (i.e., members of $D_{\langle t,t\rangle}$).

(53) Other consequences?