Quantifiers properties

Chris Potts, Ling 130a/230a: Introduction to semantics and pragmatics, Winter 2024

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1 Semantic grammar

We'll continue to work with the semantic grammar presented in our composition handout: same universe, same denotations and denotation types for everything.

2 Keenan's presentation

For Keenan (1996), quantificational determiners are functions just like ours, but he never represents the lambdas:

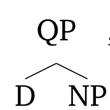
(1) Ours: $[\![every]\!] = \lambda X\Big(\lambda Y\big(\mathsf{T} \text{ if } X \subseteq Y, \text{ else } \mathsf{F}\big)\Big)$

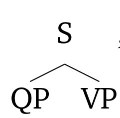
(2) Keenan's: $[\![every]\!](A)(B) = \mathsf{T}$ iff $A \subseteq B$

Keenan doesn't fully explain how the compositional semantics should work for his theory, but it's pretty clear that he has in mind something that is exactly like our theory. He writes:

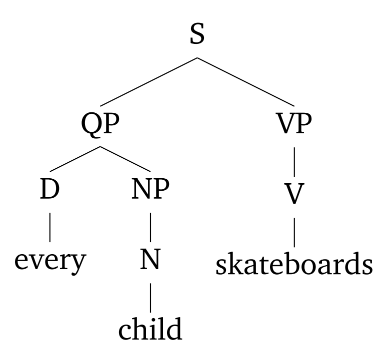
> "So we think of a Det_1 as combining with an N to make an NP, the latter combining with P_1s to make Ss." (Keenan 1996:42)

Our theory uses "D" instead of "Det", "VP" instead of "P_1", and "QP" instead of "NP", and it assumes that D combines with an NP rather than an N, and it explains what "combining" means here using Rule Q1 and Q2:

(Q1) Given a syntactic structure  , $[\![\text{QP}]\!] = [\![\text{D}]\!]([\![\text{NP}]\!])$

(Q2) Given a syntactic structure  , $[\![\text{S}]\!] = [\![\text{QP}]\!]([\![\text{VP}]\!])$

(3)



3 Some uncontroversial (?) determiner meanings

I think these meanings are pretty clear. We (like Keenan) are not attending to all the details of semantic composition by treating the phrasal ones as lexical items, but I think that's okay given our current goals.

(4) $[\![\textit{every}]\!] = \lambda X\Big(\lambda Y\Big(\mathsf{T} \text{ if } X \subseteq Y, \text{ else } \mathsf{F}\Big)\Big)$

(5) $[\![\textit{some}]\!] = \lambda X\Big(\lambda Y\Big(\mathsf{T} \text{ if } X \cap Y \neq \emptyset, \text{ else } \mathsf{F}\Big)\Big)$

(6) $[\![\textit{no}]\!] = \lambda X\Big(\lambda Y\Big(\mathsf{T} \text{ if } X \cap Y = \emptyset, \text{ else } \mathsf{F}\Big)\Big)$

(7) $[\![\textit{at least three}]\!] = \lambda X\Big(\lambda Y\Big(\mathsf{T} \text{ if } |X \cap Y| \geqslant 3, \text{ else } \mathsf{F}\Big)\Big)$

(8) $[\![\textit{at most three}]\!] = \lambda X\Big(\lambda Y\Big(\mathsf{T} \text{ if } |X \cap Y| \leqslant 3, \text{ else } \mathsf{F}\Big)\Big)$

(9) $[\![\textit{exactly three}]\!] = \lambda X\Big(\lambda Y\Big(\mathsf{T} \text{ if } |X \cap Y| = 3, \text{ else } \mathsf{F}\Big)\Big)$

(10) $[\![\textit{more than half of the}]\!] = \lambda X\Big(\lambda Y\Big(\mathsf{T} \text{ if } \frac{|X \cap Y|}{|X|} > \frac{1}{2}, \text{ else } \mathsf{F}\Big)\Big)$

(11) $[\![\textit{not every}]\!] = \lambda X\Big(\lambda Y\Big(\mathsf{T} \text{ if } X \nsubseteq Y, \text{ else } \mathsf{F}\Big)\Big)$

(12) $[\![\textit{up to 20 or more}]\!] =$

4 Lexical uncertainty

It may not be a fully settled matter in our speech community which of the following is correct:

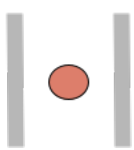
(13) $[\![$*between five and ten*$]\!]$

a. $\lambda X\Big(\lambda Y\big(\mathsf{T} \text{ if } 5 \leqslant |X \cap Y| \leqslant 10, \text{ else } \mathsf{F}\big)\Big)$

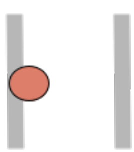
b. $\lambda X\Big(\lambda Y\big(\mathsf{T} \text{ if } 5 < |X \cap Y| < 10, \text{ else } \mathsf{F}\big)\Big)$

Ask yourself:

- If I told you that there would be between 5 and 10 quizzes in a quarter, and there were 10, would you feel that I had misrepresented things?
- What if I *reassured* you that there would be between 5 and 10 quizzes, and there were 5?
- Suppose I told you the price was between 5 and 10 dollars, and it was 10?
- Suppose I *reassured* you that the cost would between 5 and 10 dollars, and it was 5?
- Suppose I told you the price was between \$5.27 and \$10.34 dollars, and it was \$5.27 (or \$10.34)?
- If a government form asked you whether you had been in Canada in the period between August 1 and August 31, and you arrived in Toronto on August 31, what would you do?
- Is this dot between the two bars?



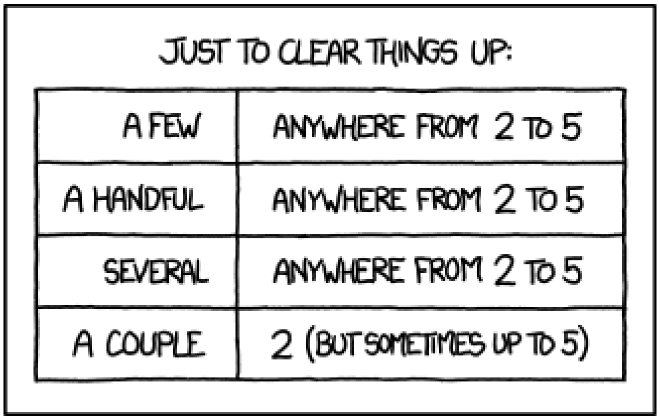
- What about this one?



People seem to be aware of this uncertainty and so sometimes add *inclusive* or *exclusive* to signal that they mean (13a) or (13b), respectively.[1]

[1]Other cases of persistent and persistently confusing lexical uncertainty include *biweekly*, *sanction*, *cleave*, and *rent*.

5 Context dependence



https://xkcd.com/1070/

I think the xkcd meanings are reasonable for *several* and *a couple*, but I am not sure whether the bounds would be included (see sec. 4):

(14) $[\![\textit{several}]\!]$

a. $\lambda X\Big(\lambda Y\big(\mathsf{T} \text{ if } 2 \leqslant |X \cap Y| \leqslant 5, \text{ else } \mathsf{F}\big)\Big)$

b. $\lambda X\Big(\lambda Y\big(\mathsf{T} \text{ if } 2 < |X \cap Y| < 5, \text{ else } \mathsf{F}\big)\Big)$

For *a few* and *a handful*, I think the number depends on the kinds of objects we're talking about. Consider *a few students*, *a few books in the library*, *a few stars* – I think our standards here could shift around. So for these I would have a *free variable* with values that can vary from context to context and that the discourse participants have to (try to) coordinate on:

(15) $[\![\textit{a few}]\!] = \lambda X\Big(\lambda Y\big(\mathsf{T} \text{ if } |X \cap Y| < j, \text{ else } \mathsf{F}\big)\Big)$

We definitely need a free variable for *few* (no 'a') and *many*:

(16) a. $[\![\textit{few}]\!] = \lambda X\Big(\lambda Y\big(\mathsf{T} \text{ if } |X \cap Y| < j, \text{ else } \mathsf{F}\big)\Big)$

b. $[\![\textit{many}]\!] = \lambda X\Big(\lambda Y\big(\mathsf{T} \text{ if } |X \cap Y| > k, \text{ else } \mathsf{F}\big)\Big)$

Some others to ponder (see Keenan's "approximative Dets"):

(17) $[\![\textit{approximately 10}]\!] = \lambda X\Big(\lambda Y\big(\mathsf{T} \text{ if } |X \cap Y| \approx 10, \text{ else } \mathsf{F}\big)\Big)$

(18) $[\![\textit{almost no}]\!] = \lambda X\Big(\lambda Y\big(\mathsf{T} \text{ if } |X \cap Y| \approx 0, \text{ else } \mathsf{F}\big)\Big)$

6 A closer look at *most*

6.1 Mark Liberman's survey

Mark Liberman noticed (19) and wrote: "I (think I) always took most to mean exactly "more than half", so Irving's "I wouldn't say 'most' but I'd say 'more than half' " took me aback."

From: `http://languagelog.ldc.upenn.edu/nll/?p=2510`

(19) Kurt Andersen: I- I read somewhere that you said that now m- most of your audience, you believe, reads you not in English. They are not only overseas but people not in the United Kingdom or Australia. It's- it's people reading in-

John Irving: I wouldn't say- I wouldn't say "most" but I'd say "more than half". Sure, more than half, definitely. I mean I- I sell more books in Germany than I do in the U.S. Uh I s- sell almost as many uh books in- in the Netherlands as I do in the- in the U.S.

Lots of readers left comments on Liberman's post articulating their assumptions about what *most* means, and he collected them in a follow-up:

`http://languagelog.ldc.upenn.edu/nll/?p=2511`

(20) I think 'most' licenses a default generalization, relative to a bunch of pragmatic factors, ...

(21) I think 'most' has a normative or qualitative sense in addition to a quantitative sense.

(22) For me too, "most" has a defeasible implicature of "much more than a majority".

(23) I would be with John Irving - 51% of a population isn't "most" but around 60-75% would be. (90% or more would be "almost all"; well, until it hit "all" at 100%; and 75-90% would be "a very large majority").

(24) "Most X are Y", to me, means a substantial majority of X are Y—certainly more than 50%-plus-1. Even two-thirds feels borderline.

(25) Most has always meant "more than half (but less than all)" to me. If there are 100 of us and I say "Most of us stayed behind" I mean between 51 and 99.

Liberman looked at some dictionaries:

(26) *OED*: modifying a plural count noun: the greatest number of; the majority of

(27) *Merriam-Webster*: the majority of

(28) *American Heritage*: in the greatest quantity, amount, measure, degree, or number: *to win the most votes*

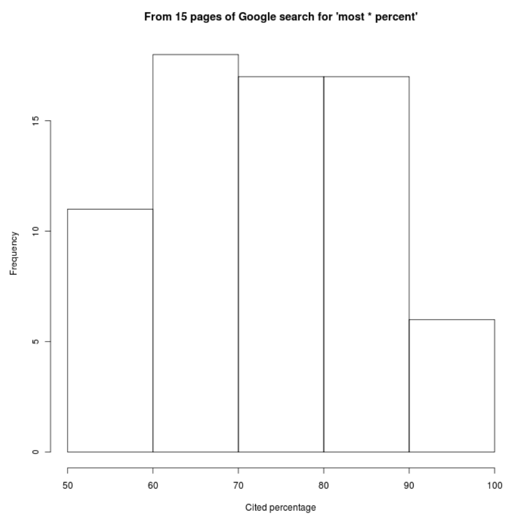
6.2 Theories

(H) $[\![most]\!] = \lambda X\Big(\lambda Y\Big(\mathsf{T} \text{ if } \frac{|X \cap Y|}{|X|} > \frac{1}{2}, \text{ else } \mathsf{F}\Big)\Big)$ ('more than half'; identical to (10))

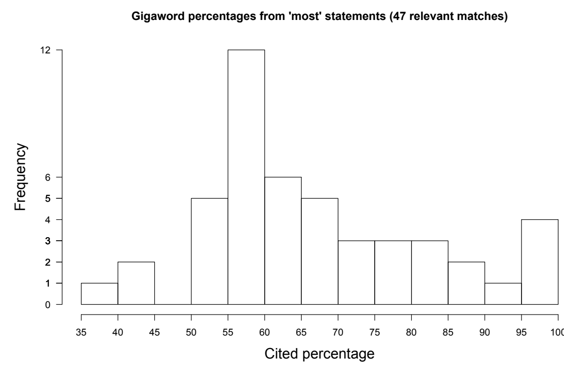
$$= \lambda X\Big(\lambda Y\Big(\mathsf{T} \text{ if } |X \cap Y| > |X - Y|, \text{ else } \mathsf{F}\Big)\Big)$$

(GH) $[\![most]\!] = \lambda X\Big(\lambda Y\Big(\mathsf{T} \text{ if } \frac{|X \cap Y|}{|X|} > k, \text{ else } \mathsf{F}\Big)\Big)$ (where $k \gg \frac{1}{2}$)

6.3 Corpus experiments



(a) Liberman's results for Googling `"most * percent"` and picking our the first 150 hits with numerical percentages.

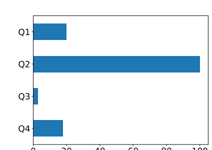


(b) My experiment using regular expressions to search the Gigaword corpus, a 1 billion word corpus of English newswire text. Link to my examples.

Liberman: "it's pretty clear that the whole range from 50.1 to 99.9 is getting some action."

Very close to an attested 100% example using *majority of* (via James Collins):

(29) Question 2 was the only one for which most participants answered yes (in fact, 100% answered yes in that case).



My three surprising cases below 50% seem to involve implicit additional restrictions:

(30) most homes (39 percent) have a separate room where the pc is

(31) found that most of them (42 percent) focus on what he dubs

(32) most of the country (42 percent) will

Liberman did an additional post giving lots of citations and abstracts for psycholinguistic and theoretical work on *most*: `http://languagelog.ldc.upenn.edu/nll/?p=2516`

6.3.1 Psycholinguistic evidence for the 50% view (H)

Pietroski et al. (2009) argue that (H) is correct in a deep sense: speakers actually calculate and then compare the cardinalities of two sets using an approximate number system, "an evolutionarily ancient piece of cognitive machinery that is shared throughout the animal kingdom and does not require explicit training with number in order to develop" (p. 565). In their experiment, they briefly showed participants arrangements of yellow and blue dots (for 200 ms) and asked them to answer 'Yes' or 'No' to the statement 'Most of the dots are blue'.

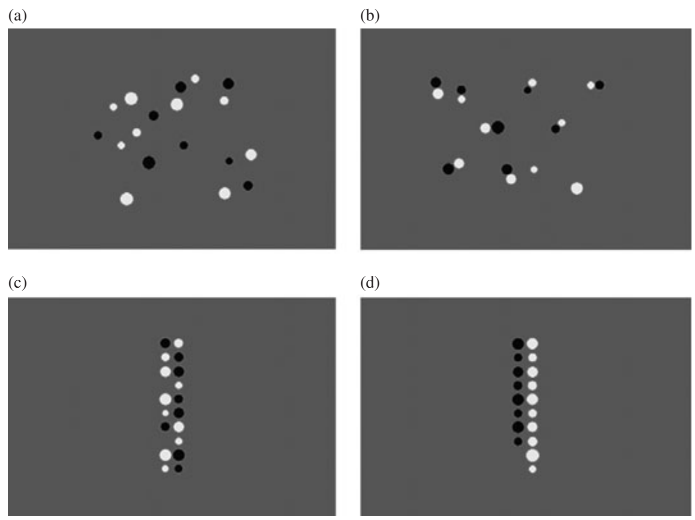


Figure 6 *Four sample trial images, from Experiment 1, in which most of the dots are yellow: (a) Scattered Random, (b) Scattered Pairs, (c) Column Pairs Mixed, (d) Column Pairs Sorted*

One of their findings is that accuracy decreases as the ratio of yellow to blue dots gets smaller. They attribute this to the approximate nature of the approximate number system. However, even when the ratio is very close to 1, subjects still answer 'Yes' correctly well above half the time:

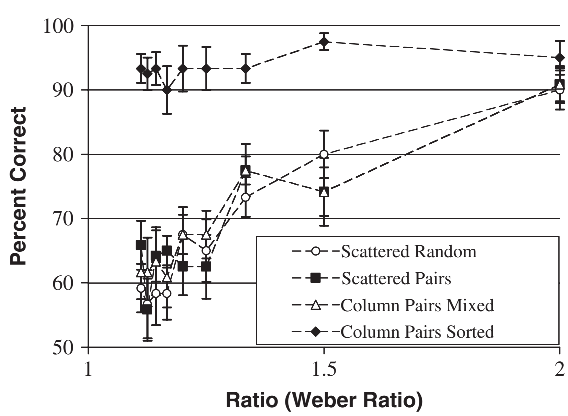
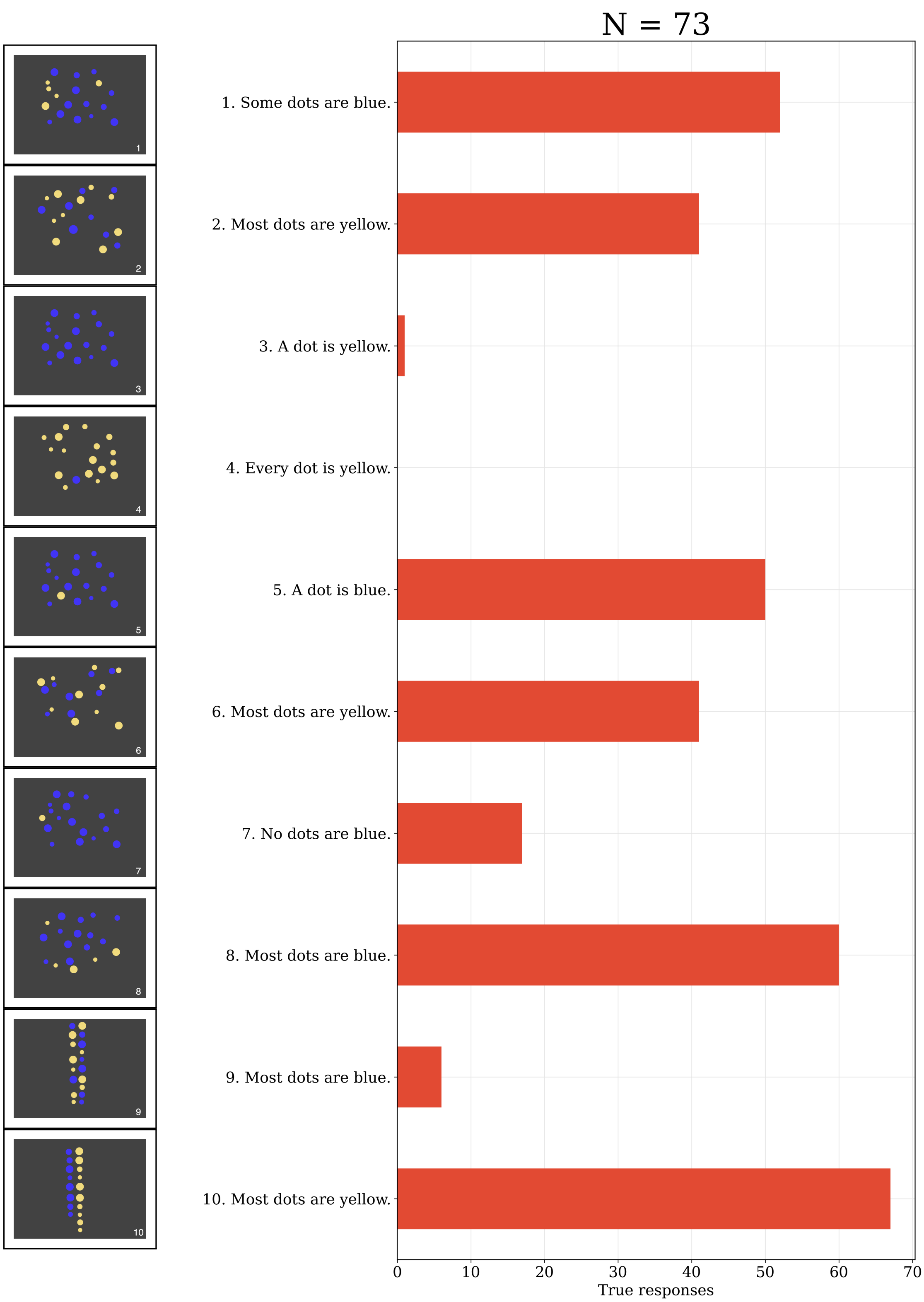


Figure 7 *Percent Correct versus Ratio (bigger # / smaller #) for the four conditions in Experiment 1*

6.4 Results of our in-class experiment



7 Quantificational determiner classes

7.1 Intersectivity

(33) A determiner D is intersective iff $D(A)(B) = D(B)(A)$ for all A and B.

(34) *some*?

(35) *every*?

(36) *no*?

(37) *at most four*?

7.2 Conservativity

(38) A determiner D is conservative iff $D(A)(B) = D(A)(A \cap B)$ for all A and B.

(39) *some*?

(40) *every*?

(41) *no*?

(42) *most*?

Proposed universal (Barwise & Cooper 1981) Every lexical determiner in every language is conservative.

Keenan (1996: 55)

> "With at most a few exceptions[1] English Dets denote conservative functions."
>
> **From Keenan's footnote 1:**
>
> "All putative counterexamples to Conservativity in the literature are ones in which a sentence of the form *Det A's are B's* is interpreted as $D(B)(A)$, where D is conservative. So the problem is not that Det fails to be conservative, rather it lies with matching the Noun and Predicate properties with the arguments of the Det denotation."

Potential counterexample: *only*

(43) Only dogs bark.

But! The evidence strongly suggests that *only* is not a determiner.

i. It can modify a wide range of constituents, not just nominals.

ii. It precedes determiner elements (e.g., *only some books*).

7.3 Monotonicity

(44) Upward monotonicity (increasing)

  a. A determiner D is upward monotone on its **first** argument iff $D(A)(B) \Rightarrow D(X)(B)$ for all A, B, X where $A \subseteq X$.

  b. A determiner D is upward monotone on its **second** argument iff $D(A)(B) \Rightarrow D(A)(X)$ for all A, B, X where $B \subseteq X$.

(45) Downward monotonicity (decreasing)

  a. A determiner D is downward monotone on its **first** argument iff $D(A)(B) \Rightarrow D(X)(B)$ for all A, B, X where $X \subseteq A$.

  b. A determiner D is downward monotone on its **second** argument iff $D(A)(B) \Rightarrow D(A)(X)$ for all A, B, X where $X \subseteq B$.

(46) A determiner D is nonmonotone on an argument iff D is neither upward nor downward monotone on that argument.

(47) *some* $\Big(\ \Uparrow \ \Big)\Big(\ \Uparrow \ \Big)$

(48) *no* $\Big(\ \Downarrow \ \Big)\Big(\ \Downarrow \ \Big)$

(49) *every* $\Big(\qquad \Big)\Big(\qquad \Big)$

(50) *at most ten* $\Big(\qquad \Big)\Big(\qquad \Big)$

(51) *exactly three* $\Big(\qquad \Big)\Big(\qquad \Big)$

(52) *most* $\Big(\qquad \Big)\Big(\qquad \Big)$

References

Barwise, Jon & Robin Cooper. 1981. Generalized quantifiers and natural language. *Linguistics and Philosophy* 4(4). 159–219.

Keenan, Edward L. 1996. The semantics of determiners. In Shalom Lappin (ed.), *The handbook of contemporary semantic theory*, 41–63. Oxford: Blackwell.

Pietroski, Paul, Jeffrey Lidz, Tim Hunter & Justin Halberda. 2009. The meaning of 'most': Semantics, numerosity and psychology. *Mind and Language* 24(5). 554–585.