Semantic composition

Chris Potts, Ling 130a/230a: Introduction to semantics and pragmatics, Winter 2024

Jan 30

1 Overview

- This handout describes our core theory of semantic composition. It's a bare-bones theory, but still powerful in the sense that generalizing it to a wider range of sentences is straightforward.
- The theory is *guaranteed compositional* even by the strictest interpretation of the principle.
- The most important conceptual move is to interpret lexical items as sets and functions. Meanings might not literally *be* sets and functions, but, following Lewis's advice, we're hypothesizing that at least these *do what meanings do*.
- We need a bunch of rules in order to respect the syntactic structures. However, there are just a few rules of semantic composition and they are very simple.
- So, in essence, if you know the lexical meanings of your language and you can put them together according to the syntactic rules, then the only other concepts you need to be a full-fledged interpreter are a few simple semantic composition rules.
- The entire grammar presented here is implemented in very simple Python code here: `https://web.stanford.edu/class/linguist130a/materials/semgrammar130a.py`

2 Notation for describing functions

2.1 Functions

IS_EVEN(x)

```
if x mod 2 = 0
    return T
else return F
```

$$\begin{bmatrix} 0 & \mapsto & \mathsf{T} \\ 1 & \mapsto & \mathsf{F} \\ 2 & \mapsto & \mathsf{T} \\ 3 & \mapsto & \mathsf{F} \\ & \vdots & \end{bmatrix}$$

$$\lambda x\,(\mathsf{T} \text{ if } x \bmod 2 = 0 \text{ else } \mathsf{F})$$

2.2 Function application

$$\text{IS\_EVEN}(1) = \mathsf{F}$$

$$\begin{bmatrix} 0 & \mapsto & \mathsf{T} \\ 1 & \mapsto & \mathsf{F} \\ 2 & \mapsto & \mathsf{T} \\ 3 & \mapsto & \mathsf{F} \\ & \vdots & \end{bmatrix}\Big(1\Big) = \mathsf{F}$$

$$\Big(\lambda x\big(\mathsf{T} \text{ if } x \bmod 2 = 0, \text{ else } \quad \mathsf{F}\big)\Big)\big(1\big)$$

$$\big(\mathsf{T} \text{ if } 1 \bmod 2 = 0, \text{ else } \quad \mathsf{F}\big)$$

$$\mathsf{F}$$

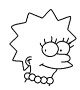
3 Basic semantic objects

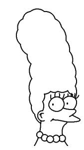
3.1 Truth values

`T` for truth and `F` for falsity.

3.2 Universe

The set of entities in our tiny possible world:

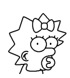
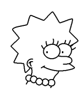
$U = \{$ , , ,  $\}$

Apologies to Marge! Her hair is so tall that she would make this handout very long! 

4 Semantic lexicon

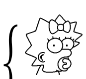
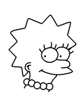
4.1 PNs

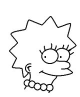
Proper names are directly referential:

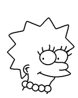
$[\![\textit{Maggie}]\!] =$  $[\![\textit{Lisa}]\!] =$  $[\![\textit{Bart}]\!] =$  $[\![\textit{Homer}]\!] =$ 

4.2 Ns

Nouns denote sets of entities (subsets of U):

(1) $[\![\textit{Simpson}]\!] = \{$ , , ,  $\}$

(2) $[\![\textit{child}]\!] = \{$ , ,  $\}$

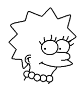
(3) $[\![\textit{student}]\!] = \{$ ,  $\}$

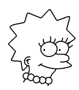
(4) $[\![\textit{parent}]\!] = \{$  $\}$

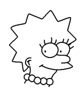
4.3 Intransitive Vs

Intransitive verbs also denote sets of entities (subsets of U):

(5) $[\![\textit{skateboards}]\!] = \{$  ,  $\}$

(6) $[\![\textit{studies}]\!] = \{$  $\}$

(7) $[\![\textit{introspects}]\!] = \{$  ,  $\}$

(8) $[\![\textit{speaks}]\!] = \{$  ,  ,  $\}$

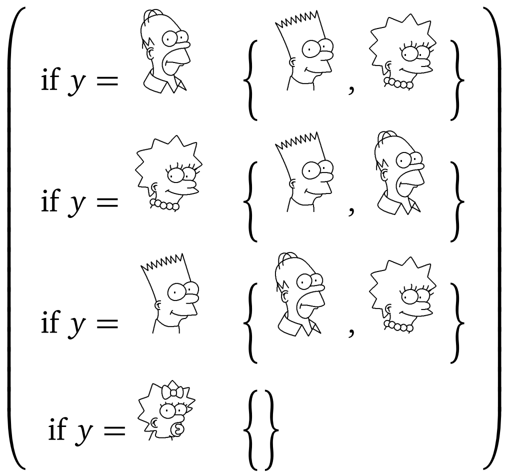
In our grammar, intransitive Vs will combine with the subject of the sentence to produce a truth-valued claim by testing whether the subject's denotation is a member of the verb's denotation.

Bart skateboards

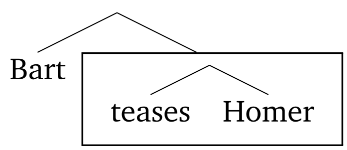
Maggie skateboards

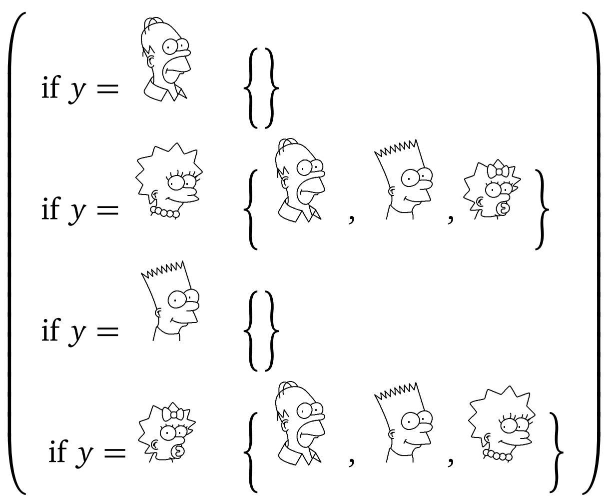
4.4 Transitive Vs

Transitive verbs denote functions from entities into sets of entities:

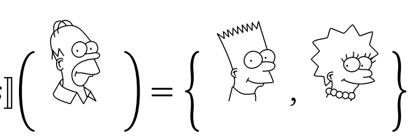
(9) $[\![\textit{teases}]\!] = \lambda y$ 

Bart, Lisa, and Homer all tease each other. Maggie neither teases nor is teased. Note: the object comes in first as y, so the return values are the people who tease y.



(10) $[\![\textit{admires}]\!] = \lambda y$ 

Everyone admires only Lisa and Maggie, but no one admires themselves.

(11) $[\![\textit{teases}(\textit{Homer})]\!] = [\![\textit{teases}]\!]$ 

(12) $[\![\textit{admires}(\textit{Maggie})]\!] =$

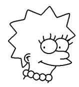
Important insight: once a transitive V combines with its object, it denotes a set of entities – semantically, it's just like an intransitive verb.

4.5 Adjectives

Adjectives combine with noun meanings to produce new noun meanings. The core of it is this very basic constituent structure:

scholarly child

You can see that we're treating the following as intersective adjectives:

(13) $[\![\textit{scholarly}]\!] = \lambda X$ ({  ,  } $\cap X$)

(14) $[\![\textit{distractible}]\!] = \lambda X$ ({  ,  } $\cap X$)

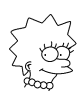
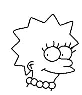
(15) $[\![\textit{hungry}]\!] = \lambda X$ ({  ,  } $\cap X$)

(16) $[\![\textit{Springfieldian}]\!] =$

The same semantic types work for the other adjective types, but they don't use $\cap$ or commit to the incoming X being true of the entities in the resulting set:

(17) $[\![\textit{alleged}]\!] = \lambda X : \{y \in U : \text{ someone claimed that } y \in X\}$

Basic semantic composition:

(18) $[\![\textit{scholarly}(\textit{child})]\!] = [\![\textit{scholarly}]\!]\big([\![\textit{child}]\!]\big) =$ {  ,  } $\cap$ {  ,  ,  }

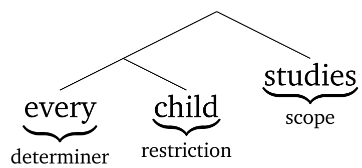
(19) $[\![\textit{hungry}\big(\textit{scholarly}(\textit{child})\big)]\!] =$

4.6 Negation

We would like a negation that operates on verb phrases like *skateboards* and *admires Maggie*:

(20) $[\![\textit{never}]\!] = \lambda X\Big(\qquad\qquad\qquad\qquad\qquad\qquad \Big)$

4.7 Quantificational determiners



(21) $[\![\textit{every}]\!] = \lambda X\Big(\lambda Y\Big(\mathsf{T} \text{ if } X \subseteq Y,\ \text{else } \mathsf{F}\Big)\Big)$

(22) $[\![\textit{every}(\textit{child})]\!] = \lambda Y\,\big(\mathsf{T} \text{ if } [\![\textit{child}]\!] \subseteq Y,\ \text{else } \mathsf{F}\big)$

(23) $[\![\textit{every}(\textit{child})(\textit{studies})]\!] = \mathsf{T} \text{ if } [\![\textit{child}]\!] \subseteq [\![\textit{studies}]\!],\ \text{else } \mathsf{F}$

(24) $[\![\textit{some}]\!] = \lambda X\Big(\lambda Y\Big(\mathsf{T} \text{ if } X \cap Y \neq \emptyset,\ \text{else } \mathsf{F}\Big)\Big)$

(25) $[\![\textit{no}]\!] = \lambda X\Big(\lambda Y\Big(\mathsf{T} \text{ if } X \cap Y = \emptyset,\ \text{else } \mathsf{F}\Big)\Big)$

(26) $[\![\textit{at least three}]\!] = \lambda X\Big(\lambda Y\Big(\mathsf{T} \text{ if } |X \cap Y| \geqslant 3,\ \text{else } \mathsf{F}\Big)\Big)$

(27) $[\![\textit{at most three}]\!] = \lambda X\Big(\lambda Y\Big(\mathsf{T} \text{ if } \qquad\qquad\qquad\qquad\qquad\qquad\qquad ,\ \text{else } \mathsf{F}\Big)\Big)$

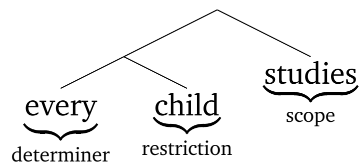
(28) $[\![\textit{most}]\!] = \lambda X\Big(\lambda Y\Big(\mathsf{T} \text{ if } \frac{|X \cap Y|}{|X|} > \frac{1}{2},\ \text{else } \mathsf{F}\Big)\Big)$

(29) $[\![\textit{between five and ten}]\!] =$

An imagined dialogue about quantificational determiners

(30) **You defined quantificational determiners as denoting relations between sets of entities. Isn't that too complicated?**

It is complicated, but it's not *too* complicated! It's the least complicated thing we could do! We really need the determiner to control both its restriction and its scope:



(31) **But couldn't *every* just denote the universe U?**

No way! We need to consider the role of the restriction: *every child*, *every scholarly parent*, and so forth.

(32) **Ok, then let's say that *every student* picks out the set of students, and *every parent* the set of parents, and so forth. That would at least be somewhat simpler.**

That still won't work! Suppose ⟦*every student*⟧ was the set of students, for example. What would we do about the verb phrase? We need ⟦*every student skateboards*⟧ to be false and ⟦*every student speaks*⟧ to be true. What are the criteria for making that distinction?

(33) **The criteria could be subset, as you gave it. ⟦*every student skateboards*⟧ is F because the set of students is not a subset of the set of skateboarders, but ⟦*every student speaks*⟧ is T because the set of students is a subset of the set of things that speak. That's just like "if x is a student, then x skateboards". It seems intuitively correct.**

Exactly! But that's just a rephrasing of the analysis we gave. You start with

$$\mathsf{T} \text{ if } [\![\textit{student}]\!] \subseteq Y, \text{ else } \mathsf{F}$$

We explicitly bind the variable Y, as in

$$\lambda Y\left(\mathsf{T} \text{ if } [\![\textit{student}]\!] \subseteq Y, \text{ else } \mathsf{F}\right)$$

This is intuitively a set of sets. That captures the variation we just noted in truth values for different verb phrases. And this is the meaning of *every student*. To get all the way back to ⟦*every*⟧, we just bind the slot filled by ⟦*student*⟧:

$$\lambda X\left(\lambda Y\left(\mathsf{T} \text{ if } X \subseteq Y, \text{ else } \mathsf{F}\right)\right)$$

This is what's in (21).

(34) **Okay, you convinced me for *every*. But surely *no*, *no student*, etc., can all just denote the empty set. That seems intuitively like what *no* means: nothingness.**

No, that won't work! Consider *no parent studies*. This is true in our possible world, but neither ⟦*parent*⟧ nor ⟦*studies*⟧ is the empty set in our possible world. It's their *intersection* that is empty if this sentence is true. And we want to say that *in general*, and that's what our theory does. We can again start with a specific claim:

$$\mathsf{T} \text{ if } [\![parent]\!] \cap [\![studies]\!] = \emptyset, \text{ else } \mathsf{F}$$

And then we back off to get $[\![no\ parent]\!]$:

$$\lambda Y\Big(\mathsf{T} \text{ if } [\![parent]\!] \cap Y = \emptyset, \text{ else } \mathsf{F}\Big)$$

And once more to get the meaning we defined in (25):

$$\lambda X\Bigg(\lambda Y\Big(\mathsf{T} \text{ if } X \cap Y = \emptyset, \text{ else } \mathsf{F}\Big)\Bigg)$$

(35) **I am starting to see that this is the least complicated thing we can do. And I also see that this basic set-up can work for lots of determiners. We start with our framework**

$$\lambda X\Bigg(\lambda Y\Big(\mathsf{T} \text{ if } \qquad\qquad\qquad\qquad\qquad\qquad , \text{ else } \mathsf{F}\Big)\Bigg)$$

and then we just need to specify what the relation is for any given determiner.

Yes!

(36) **And, if I want to, I can start with a specific instance of what I want to capture, like**

$$[\![most\ students\ skateboard]\!] = \mathsf{T} \text{ if } \frac{|[\![student]\!] \cap [\![skateboards]\!]|}{|[\![student]\!]|} > \frac{1}{2}, \text{ else } \mathsf{F}$$

and then just back out the variables with lambda binders, first for the scope:

$$[\![most\ students]\!] = \lambda Y\Big(\mathsf{T} \text{ if } \frac{|[\![student]\!] \cap Y|}{|[\![student]\!]|} > \frac{1}{2}, \text{ else } \mathsf{F}\Big)$$

and then for the restriction:

$$\lambda X\Bigg(\lambda Y\Big(\mathsf{T} \text{ if } \frac{|X \cap Y|}{|X|} > \frac{1}{2}, \text{ else } \mathsf{F}\Big)\Bigg)$$

Beautiful! And remember to always do the binding in the order you did it: restriction outer (comes in first) and scope inside (comes in second). Some determiners are order-sensitive, like *most* and *every*.

5 Semantic grammar

(Lex) Given a leaf node X, $[\![X]\!]$ is looked up in the lexicon.

(NB) Given a syntactic structure $\begin{array}{c} \mathrm{X} \\ | \\ \mathrm{Y} \end{array}$, $[\![X]\!] = [\![Y]\!]$

(S) Given a syntactic structure $\begin{array}{c} \mathrm{S} \\ \mathrm{PN} \quad \mathrm{VP} \end{array}$, $[\![S]\!] = \mathsf{T}$ if $[\![PN]\!] \in [\![VP]\!]$, else F

(A) Given a syntactic structure $\begin{array}{c} \mathrm{NP}_j \\ \mathrm{AP} \quad \mathrm{NP}_i \end{array}$, $[\![\mathrm{NP}_j]\!] = [\![\mathrm{AP}]\!]([\![\mathrm{NP}_i]\!])$

(N) Given a syntactic structure $\begin{array}{c} \mathrm{VP}_j \\ \textit{never} \quad \mathrm{VP}_i \end{array}$, $[\![\mathrm{VP}_j]\!] = [\![\textit{never}]\!]([\![\mathrm{VP}_i]\!])$

(TV) Given a syntactic structure $\begin{array}{c} \mathrm{VP} \\ \mathrm{V} \quad \mathrm{PN} \end{array}$, $[\![\mathrm{VP}]\!] = [\![\mathrm{V}]\!]([\![\mathrm{PN}]\!])$

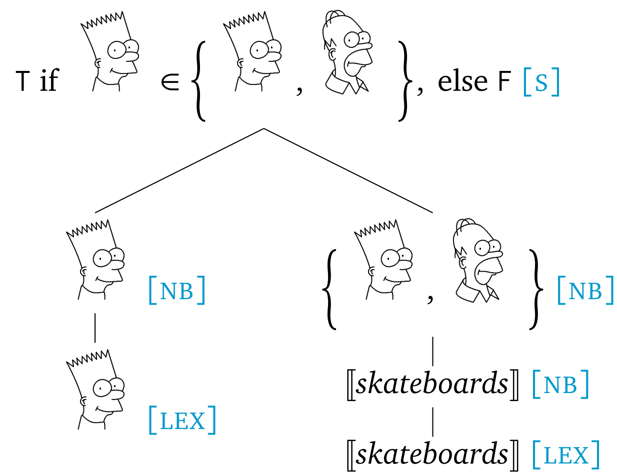
(Q1) Given a syntactic structure $\begin{array}{c} \mathrm{QP} \\ \mathrm{D} \quad \mathrm{NP} \end{array}$, $[\![\mathrm{QP}]\!] = [\![\mathrm{D}]\!]([\![\mathrm{NP}]\!])$

(Q2) Given a syntactic structure $\begin{array}{c} \mathrm{S} \\ \mathrm{QP} \quad \mathrm{VP} \end{array}$, $[\![\mathrm{S}]\!] = [\![\mathrm{QP}]\!]([\![\mathrm{VP}]\!])$

6 Illustrations

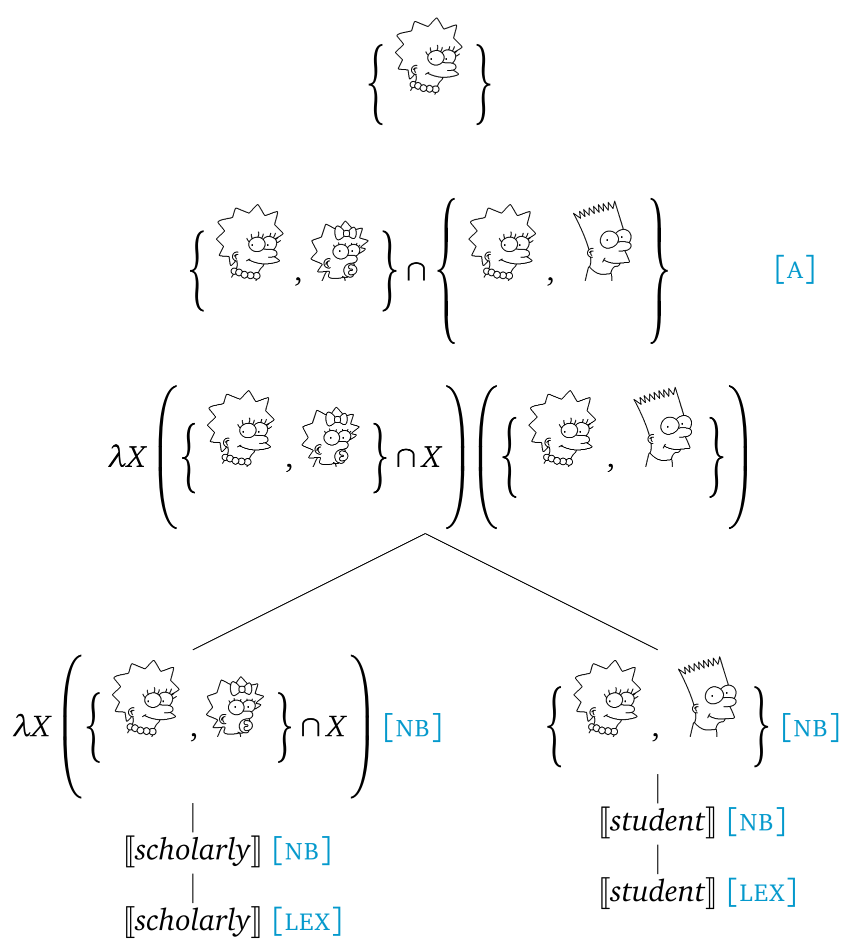
(37)

S
PN VP
Bart V
skateboards

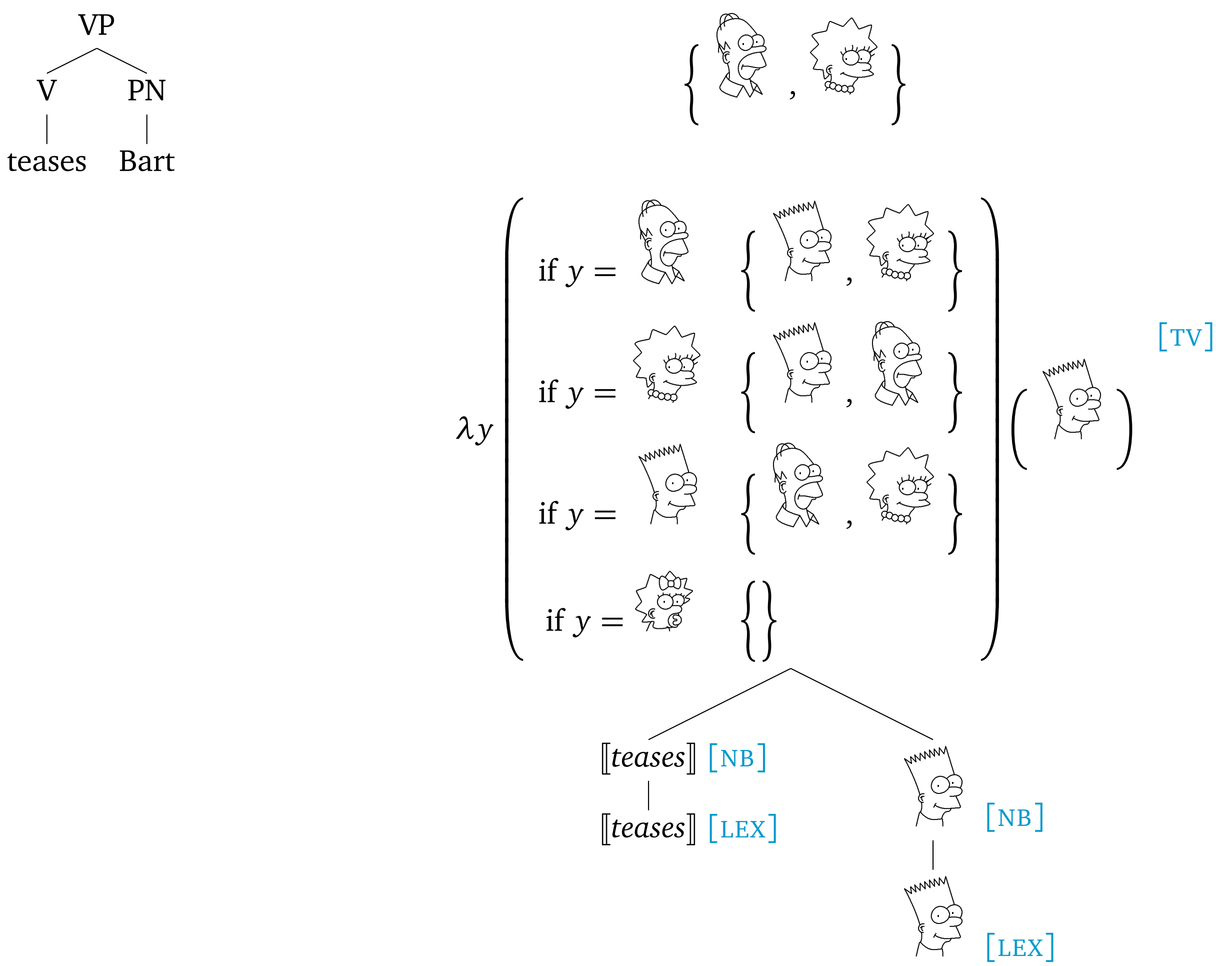


(38)

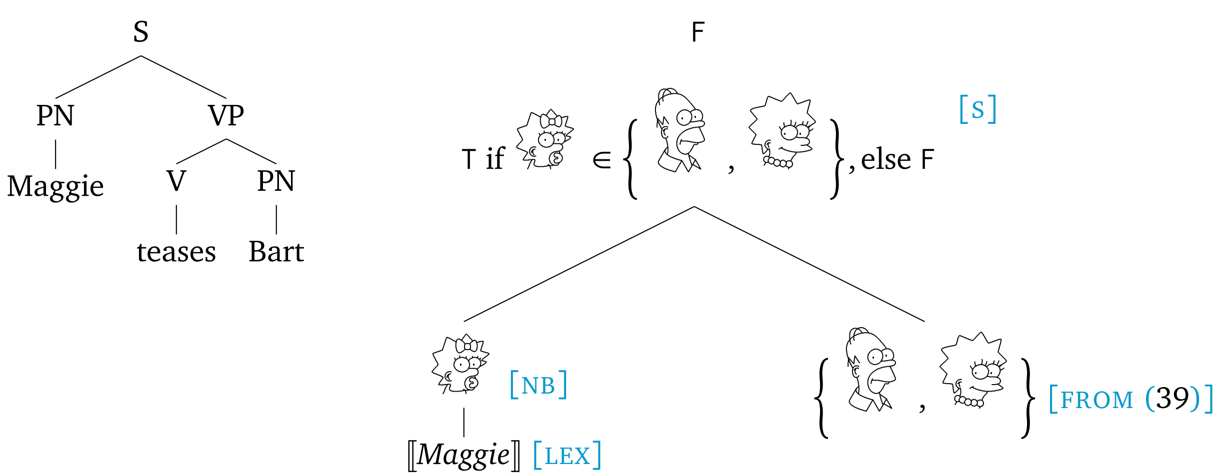
NP
AP NP
A N
scholarly student



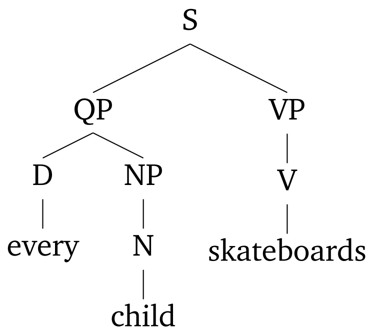
(39)

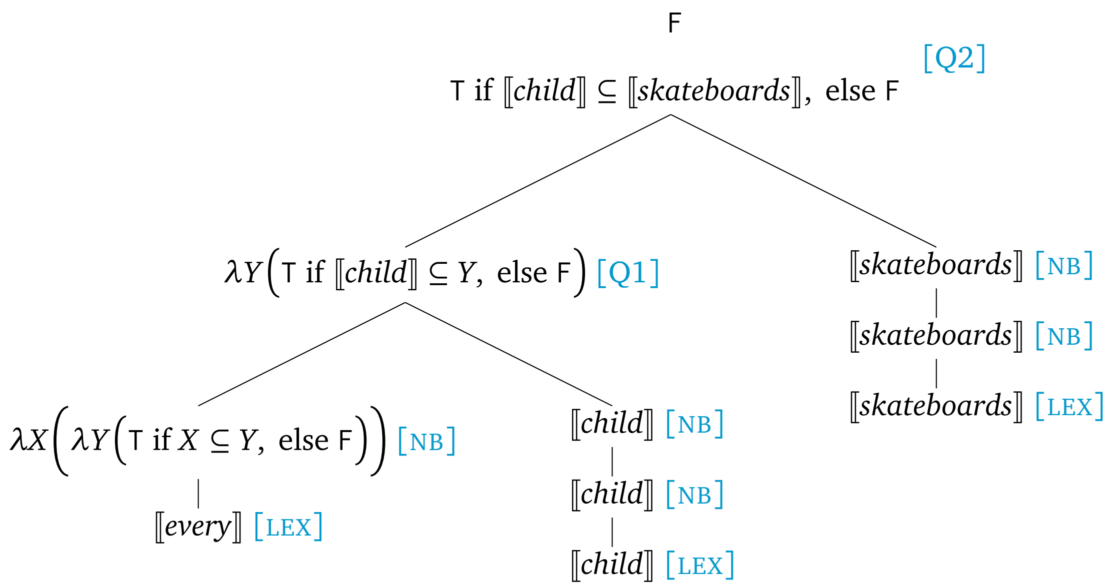

VP
V
PN
teases
Bart
λy
if y =
if y =
if y =
if y =
[TV]
⟦teases⟧ [NB]
⟦teases⟧ [LEX]
[NB]
[LEX]

(40)

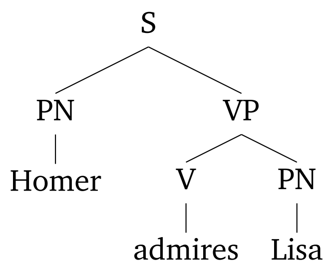

S
PN
VP
Maggie
V
PN
teases
Bart
F
T if ∈ , else F
[S]
[NB]
⟦Maggie⟧ [LEX]
[FROM (39)]

(41)

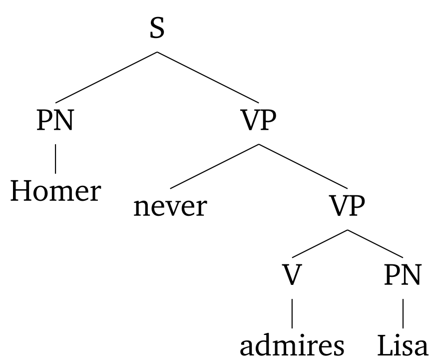

S
QP
VP
D
NP
V
every
N
skateboards
child


F
T if ⟦child⟧ ⊆ ⟦skateboards⟧, else F [Q2]
λY(T if ⟦child⟧ ⊆ Y, else F) [Q1]
⟦skateboards⟧ [NB]
⟦skateboards⟧ [NB]
⟦skateboards⟧ [LEX]
λX(λY(T if X ⊆ Y, else F)) [NB]
⟦every⟧ [LEX]
⟦child⟧ [NB]
⟦child⟧ [NB]
⟦child⟧ [LEX]

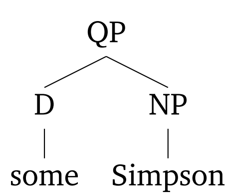
(42)


S
PN
VP
Homer
V
PN
admires
Lisa

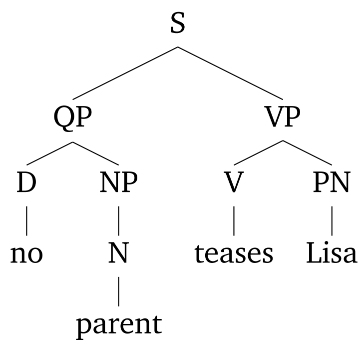
(43)


S
PN
VP
Homer
never
VP
V
PN
admires
Lisa

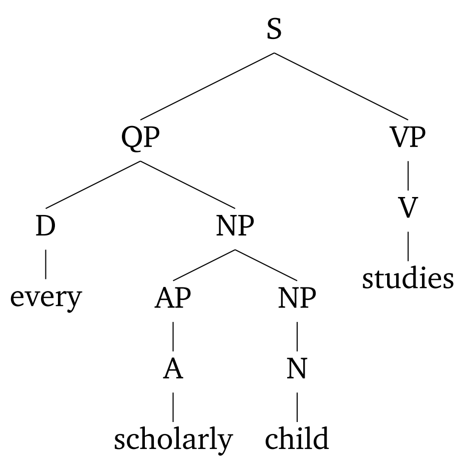
(44)


QP
D
NP
some
Simpson

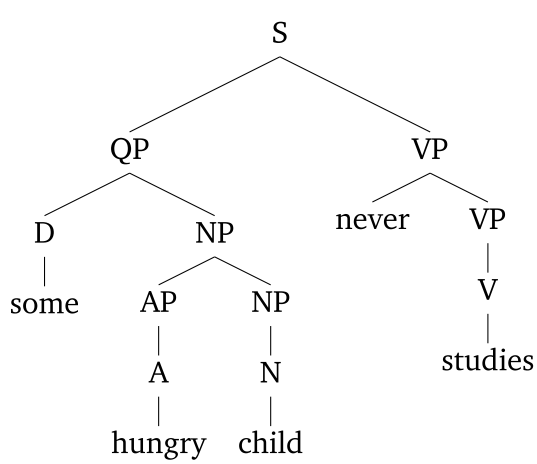
(45)


S
QP
VP
D
NP
V
PN
no
N
teases
Lisa
parent

(46)


S
QP
VP
V
D
NP
studies
every
AP
NP
A
N
scholarly
child

(47)


S
QP
VP
D
NP
never
VP
some
AP
NP
V
A
N
studies
hungry
child